Markets in the Mind

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University of Warwick

Rebuilding Macroeconomics Workshop
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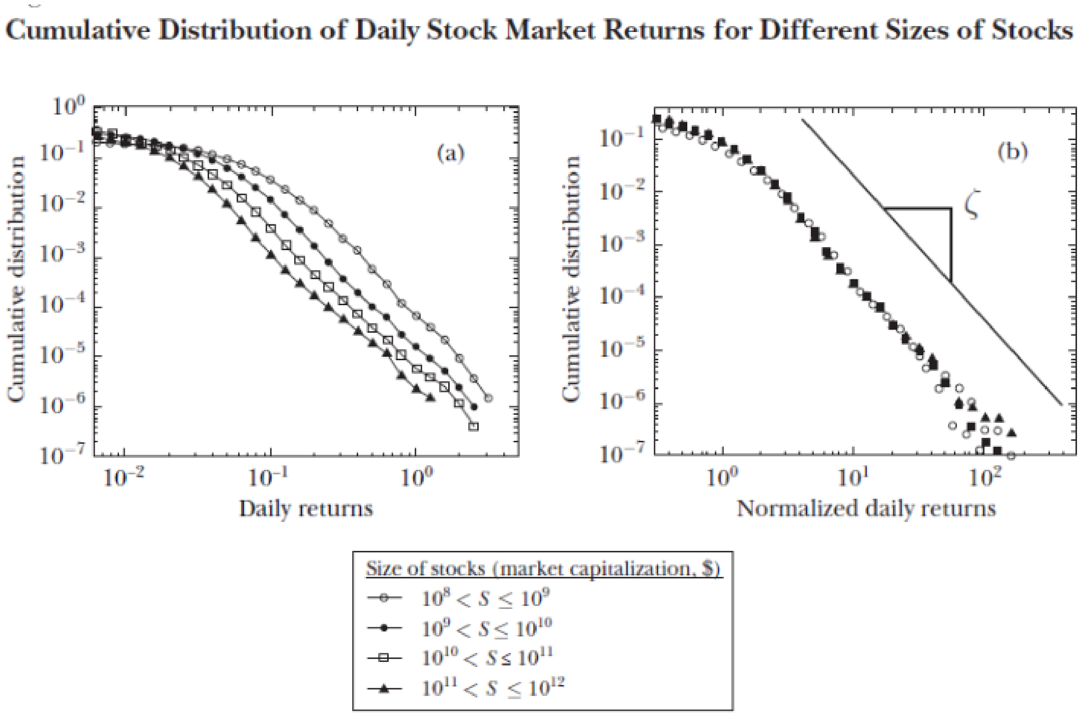
Scale Invariance in Financial Markets

Financial markets show notable fractal patterns, with clusters of activity and sudden large movements in price



The Stylised Statistical Properties of Financial Markets

Statistical facts summarized by Cont (2001):

- Absence of autocorrelation in return
- Heavy tails (conditional and unconditional)
- Aggregational Gaussianity
- Intermittency
- Volatility clustering
- Slow decay of autocorrelation in absolute returns

- Gain/loss asymmetry
- Leverage effect
- Volume/volatility correlation
- Asymmetry in time scales

The Origins of Statistical Properties of Financial Markets

Statistical models:

- GARCH-type models [Bollerslev et al., 1992; Engle, 1995; Ding, Granger et al., 1993]
- Fractal Brownian model (i.e., self-similar process) [Mandelbrot & Van Ness, 1968]

"Statistical analysis alone is not likely to provide a definite answer for empirical phenomena in financial market, unless economic mechanisms are proposed to understand the origin of such phenomena"
Cont (2007)

Proposed economic mechanisms:

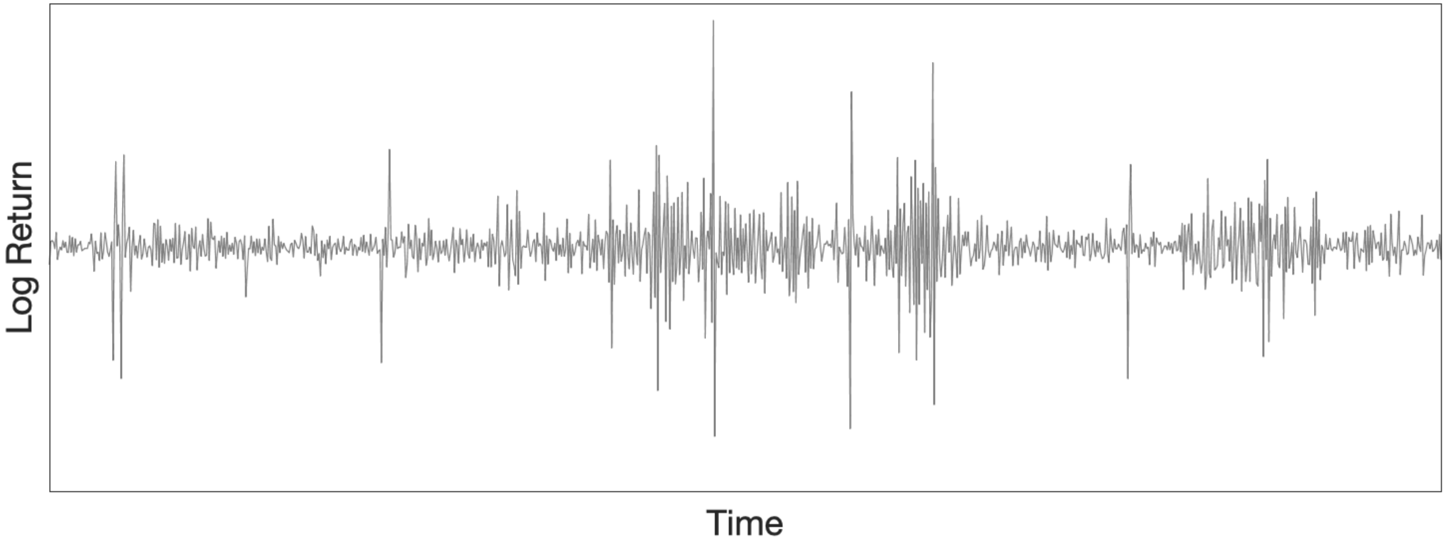
- Efficient market (random walk) hypothesis [Bachelier, 1900; Samuleson, 1965; Fama, 1970]
- Behavioural switching models [Kirman, 1993; Lux & Marchesi, 2000]
- Investor inertia to news [Cont et al., 2004]

Psychological roles in economics:

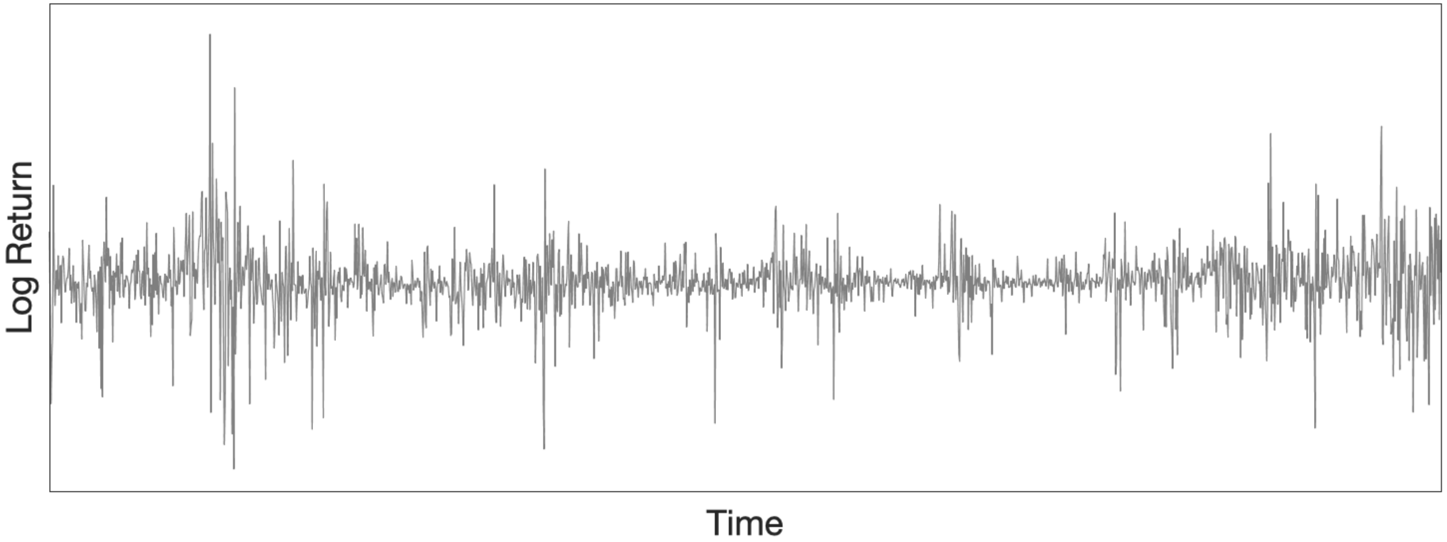
- Whether price variations reflect cognitive fluctuations in beliefs?

Can you detect the financial time series?

Tapping task



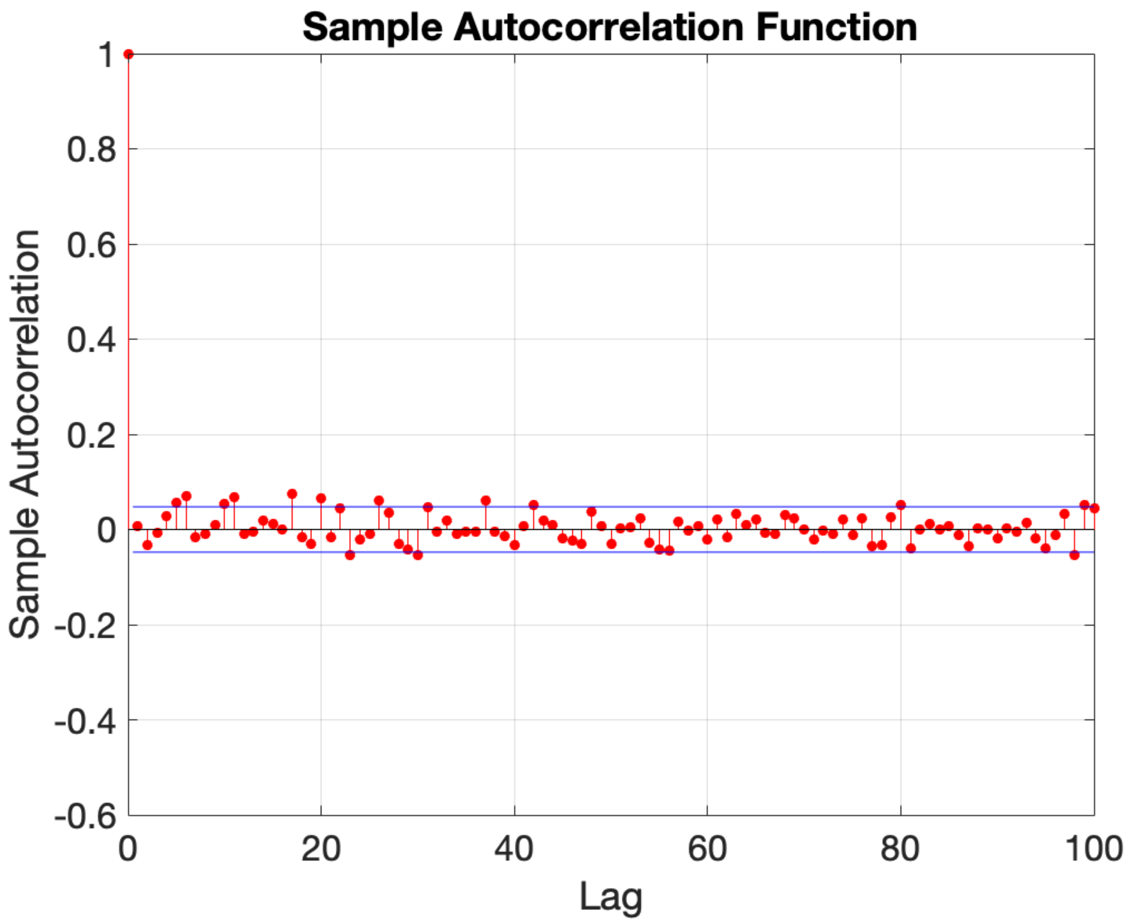
Bitcoin/USD exchange rate



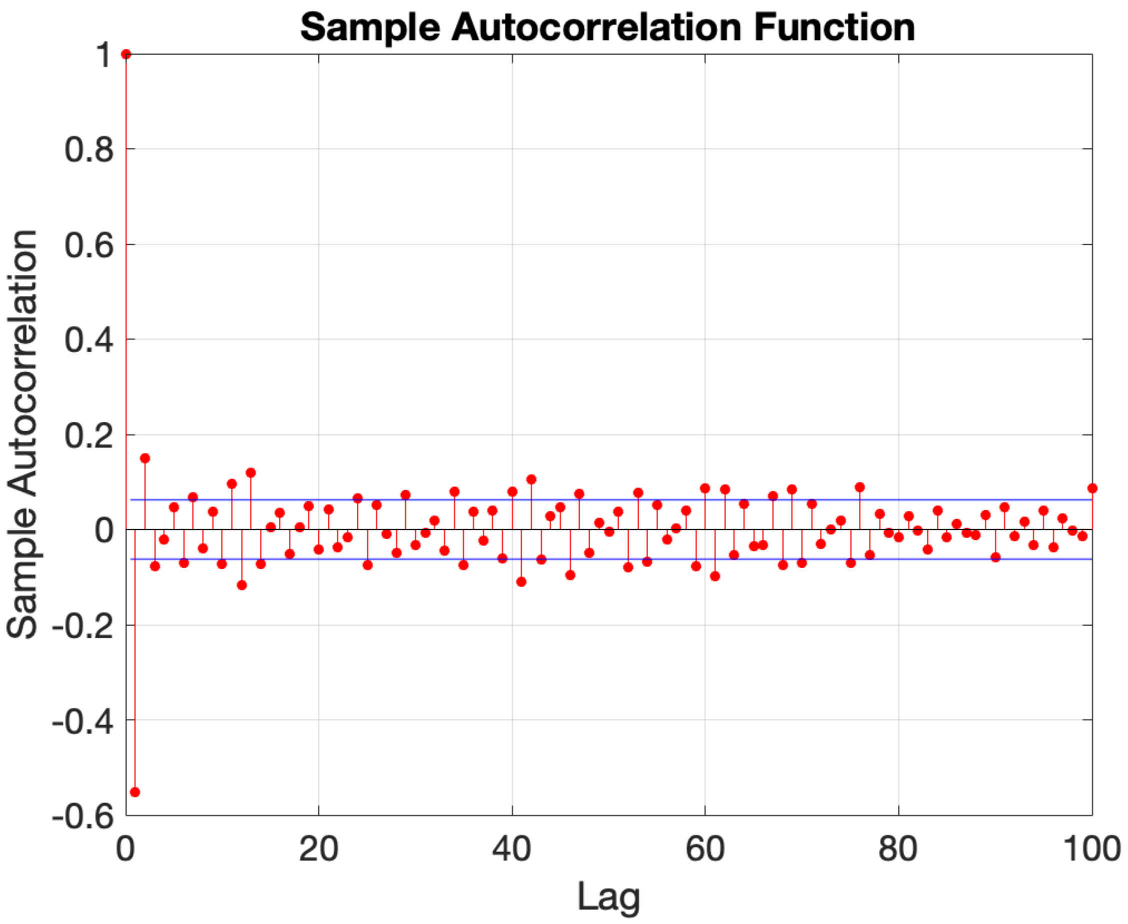
Congruent properties of financial time series and tapping task

[1] Absence of autocorrelation in asset returns

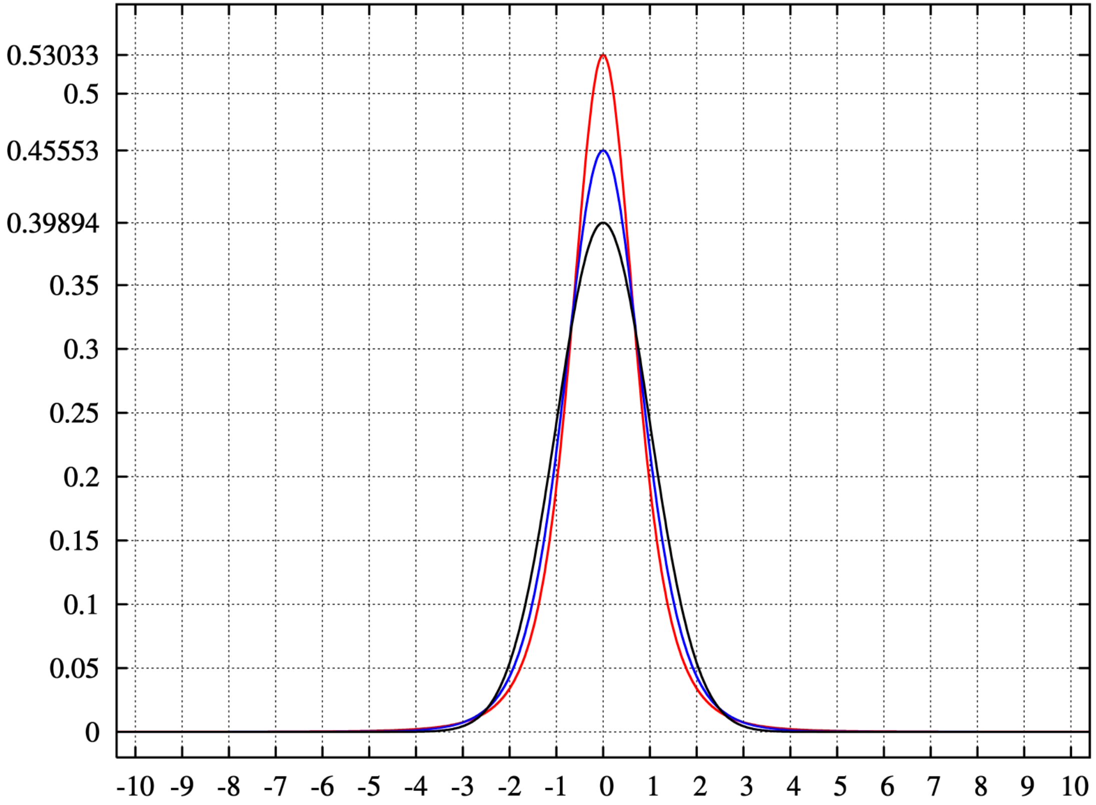
Bitcoin/USD Exchange Rate

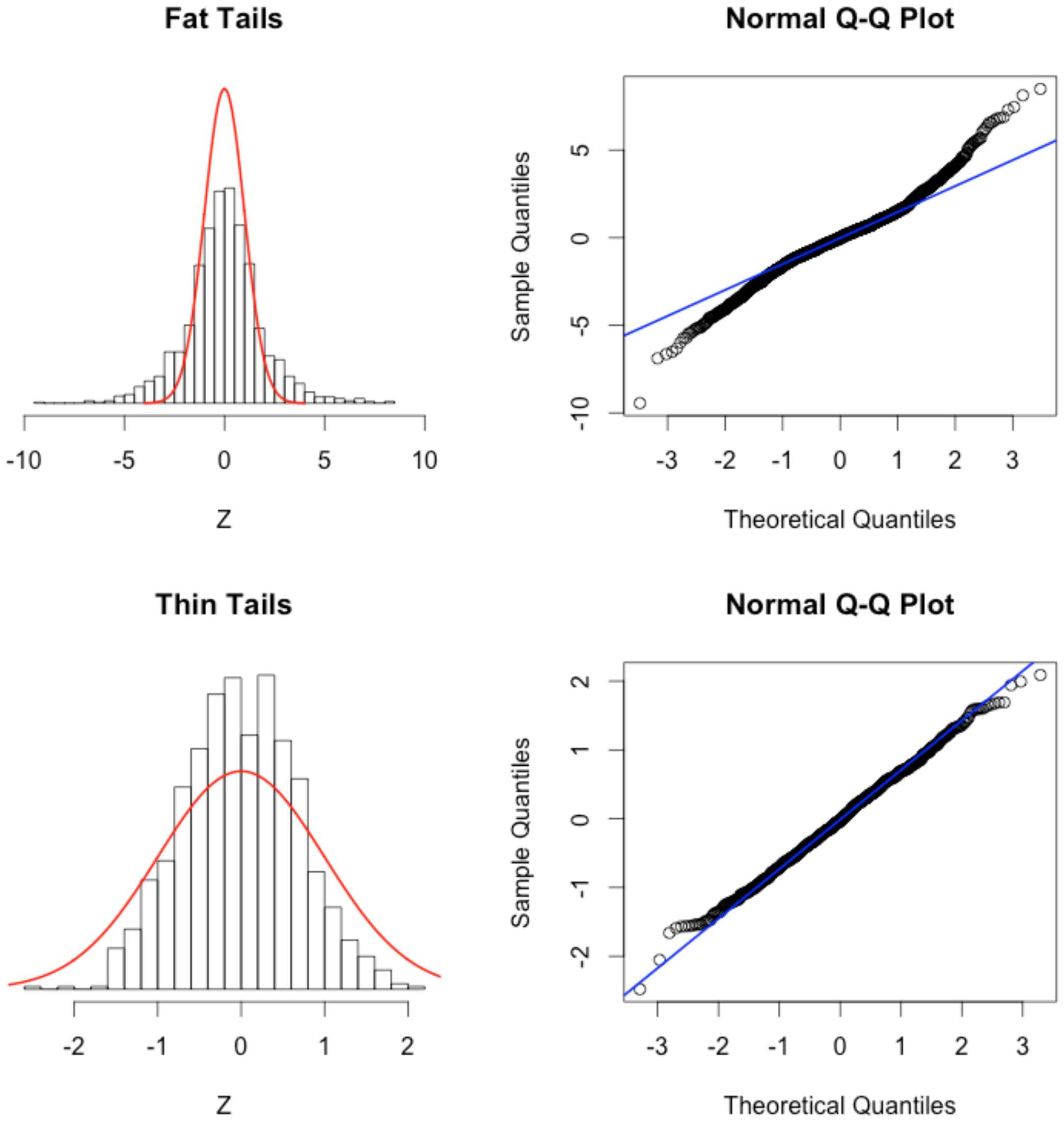


Tapping task



Kurtosis: Model-free Estimation of Tailedness



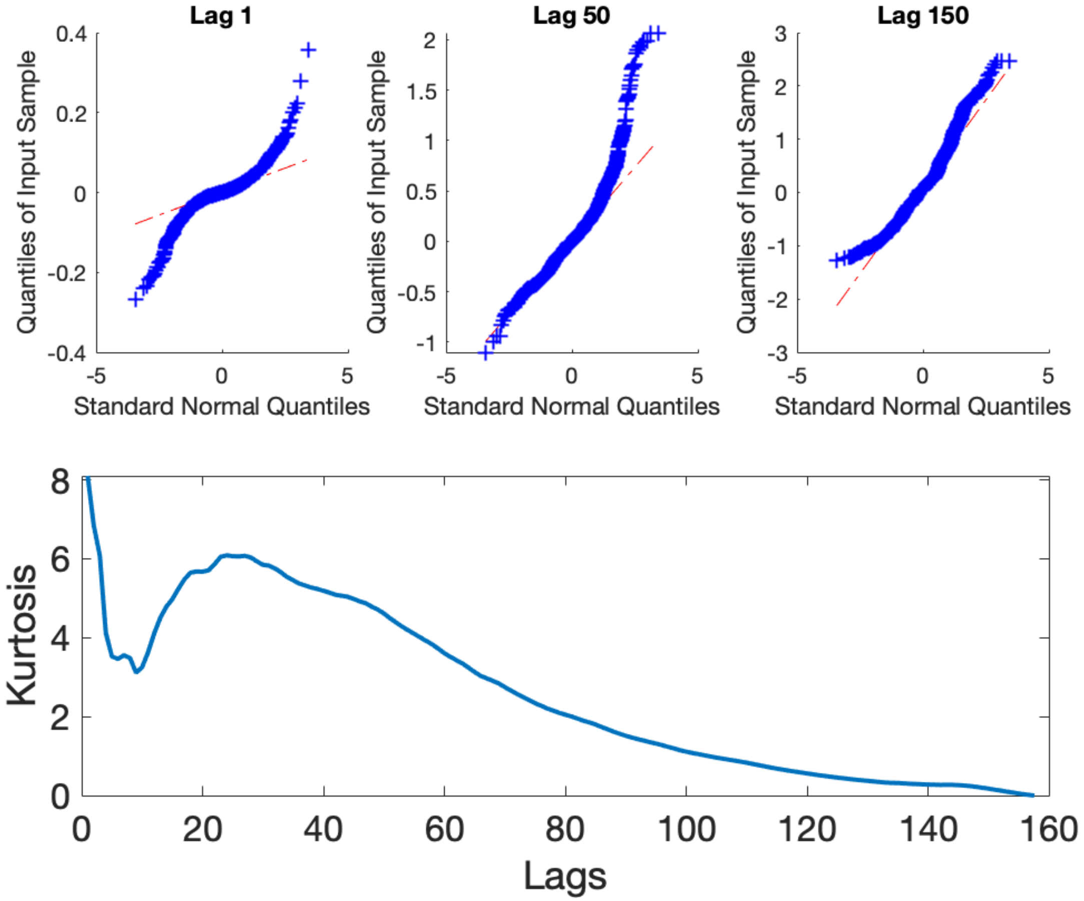


Congruent properties of financial time series and tapping task

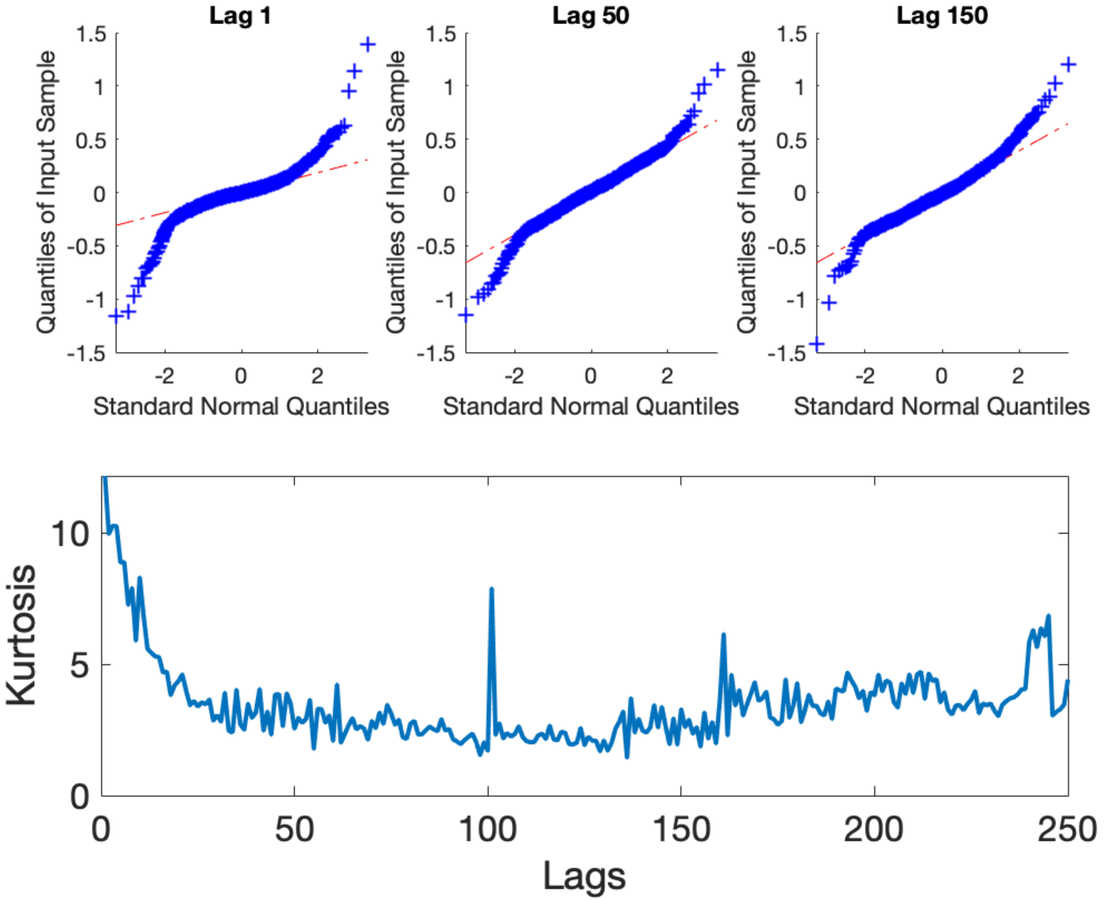
[2] Unconditional heavy tails

[4] Aggregational Gaussianity

Bitcoin/USD Exchange Rate



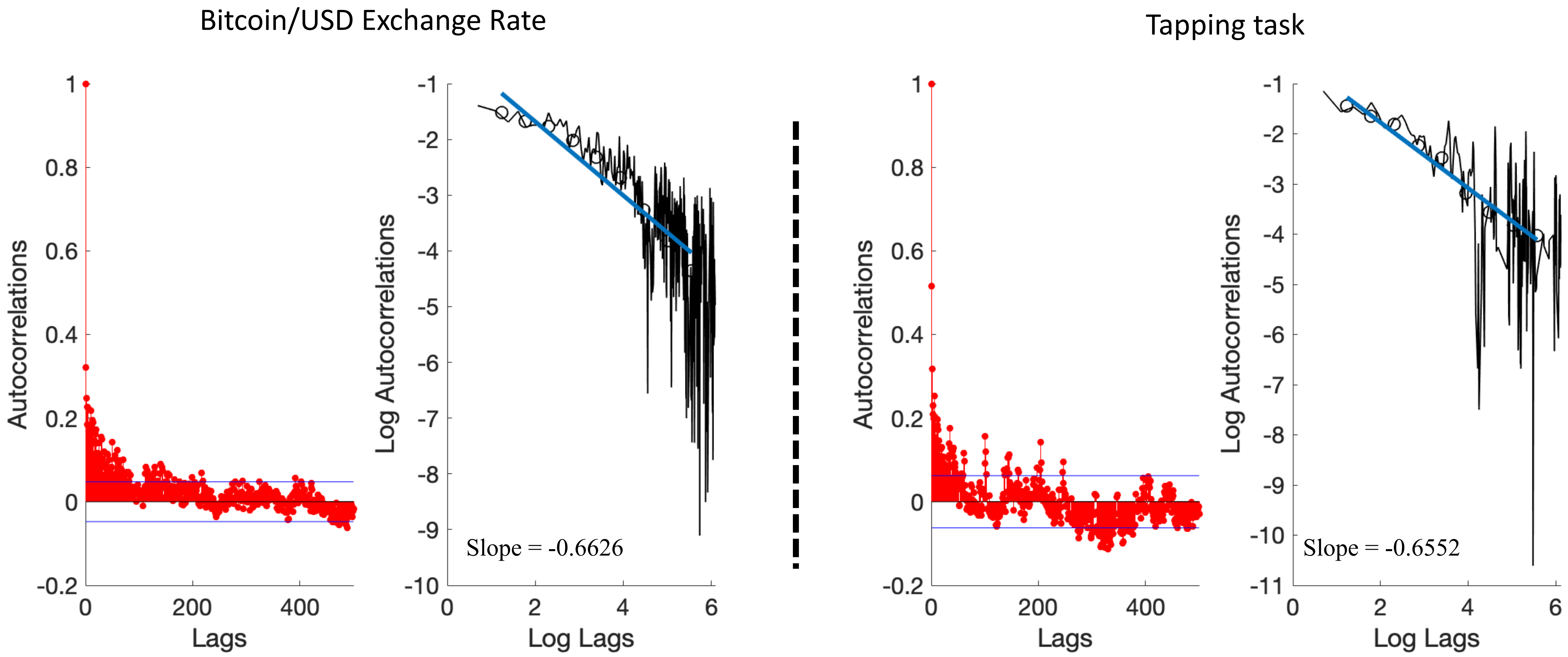
Tapping task



Congruent properties of financial time series and tapping task

[6] Volatility clustering

[8] Slow decay of autocorrelation in absolute returns

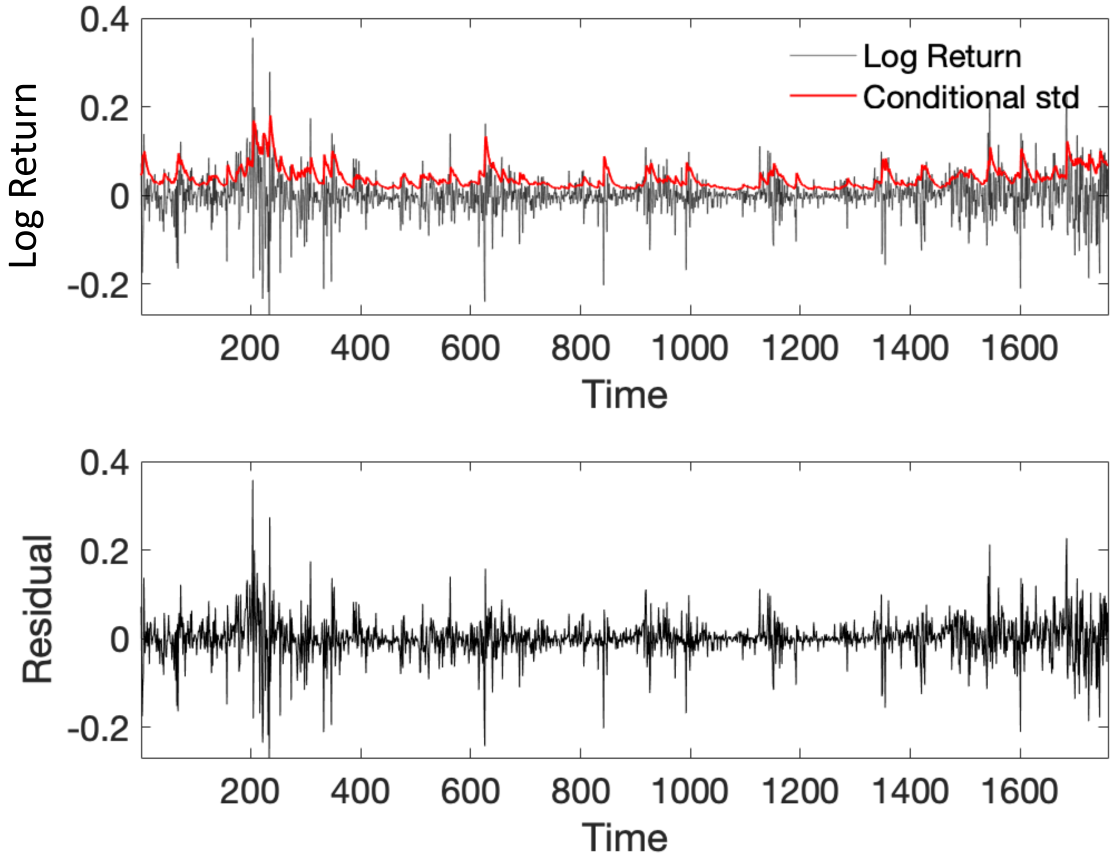


Congruent properties of financial time series and tapping task

[5] Intermittency

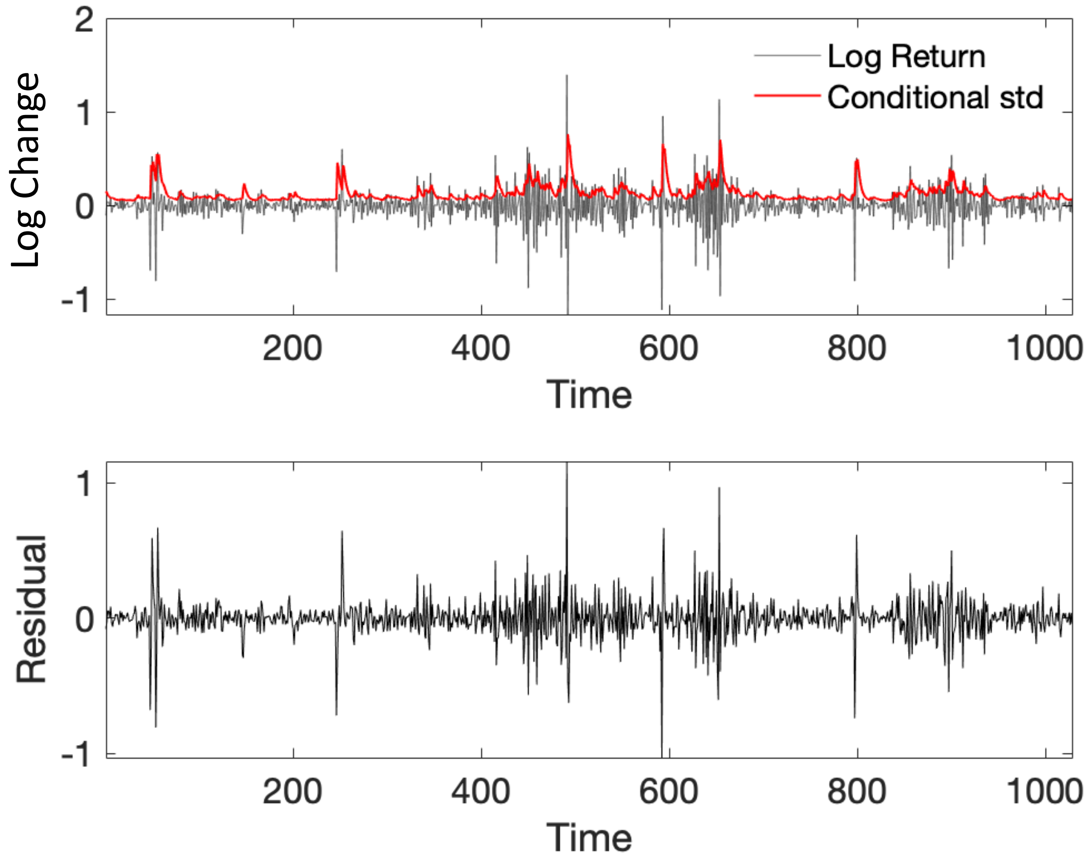
[6] Volatility clustering

Bitcoin/USD Exchange Rate



GARCH(1) = 0.8191 (p<0.0001)

Tapping task

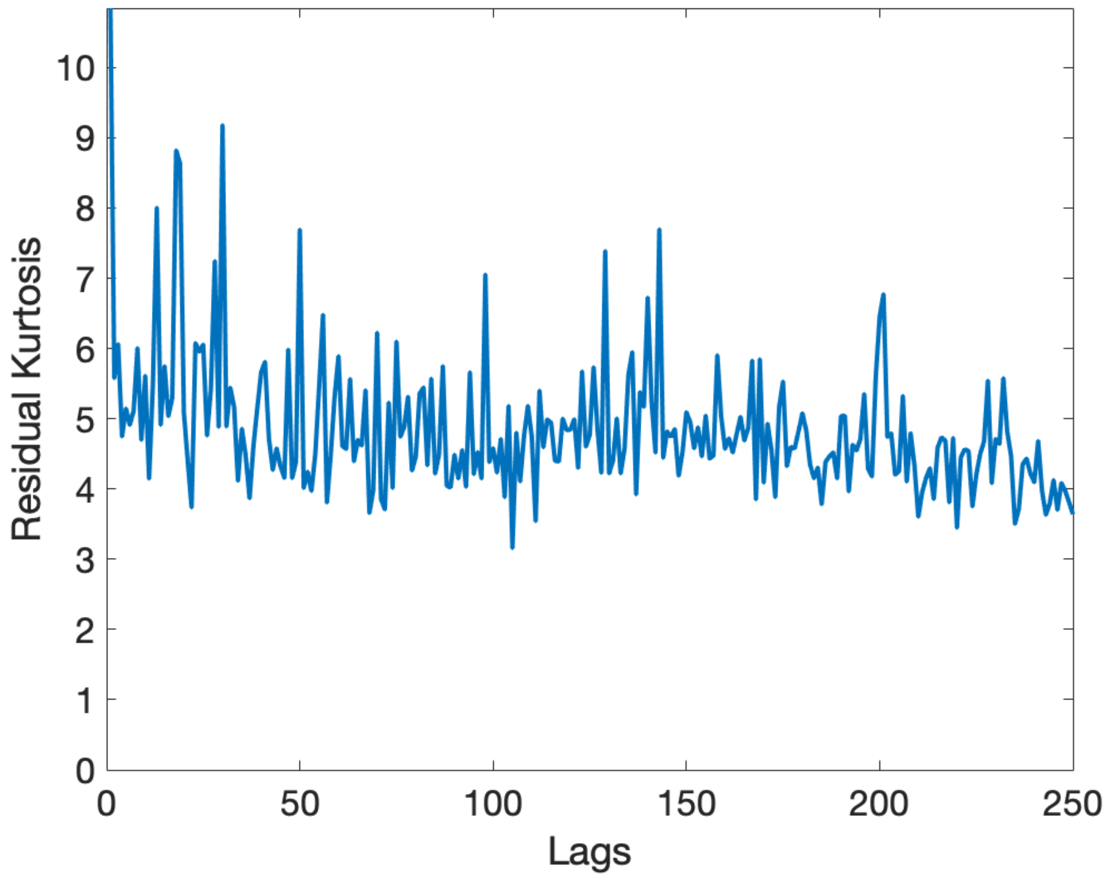


GARCH(1) = 0.6026 (p<0.0001)

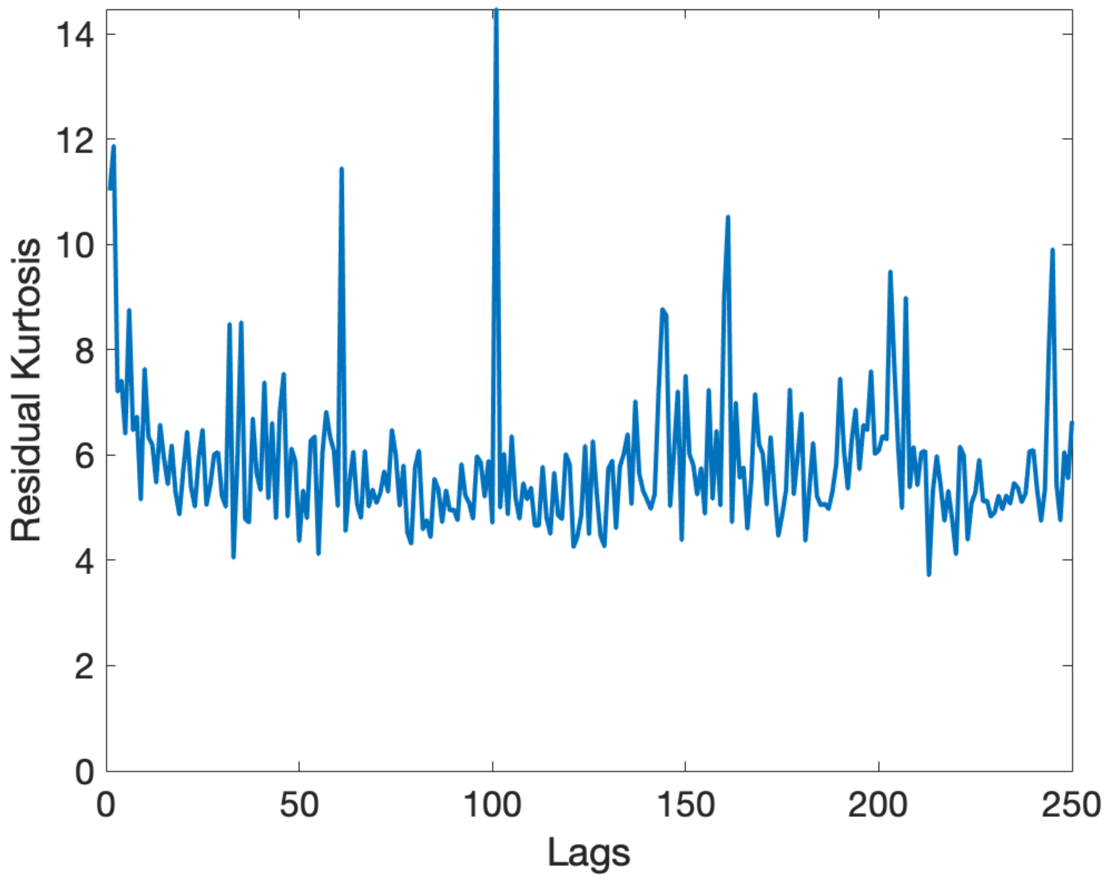
Congruent properties of financial time series and tapping task

[7] Conditional heavy tails

Bitcoin/USD Exchange Rate



Tapping task



Comparing Individual Decisions and Market Movements

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- Individual decisions in a simple time estimation task bear a striking resemblance to financial market movements

- We therefore want to investigate whether these behaviours may share a common origin in decision making

- Sample-based approximation to Bayesian inference (e.g., rational expectation)

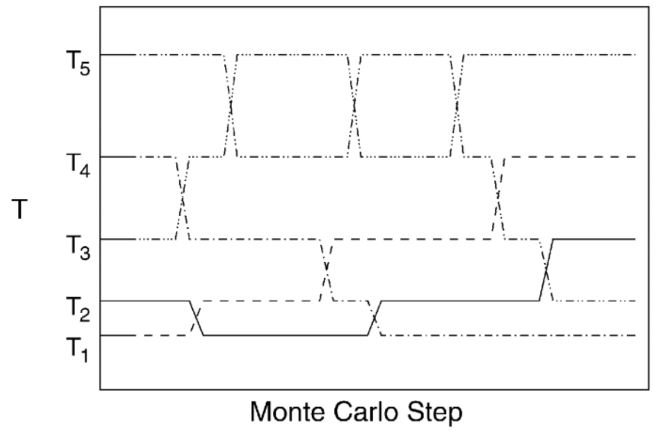
- Here, we focus on one algorithm in particular which has been successful in explaining the previously shown tapping data: Metropolis-Coupled Markov Chain Monte Carlo (MC^3)

Metropolis-coupled Markov Chain Monte Carlo (MC^3)

aka parallel tempering, replica-exchange MCMC

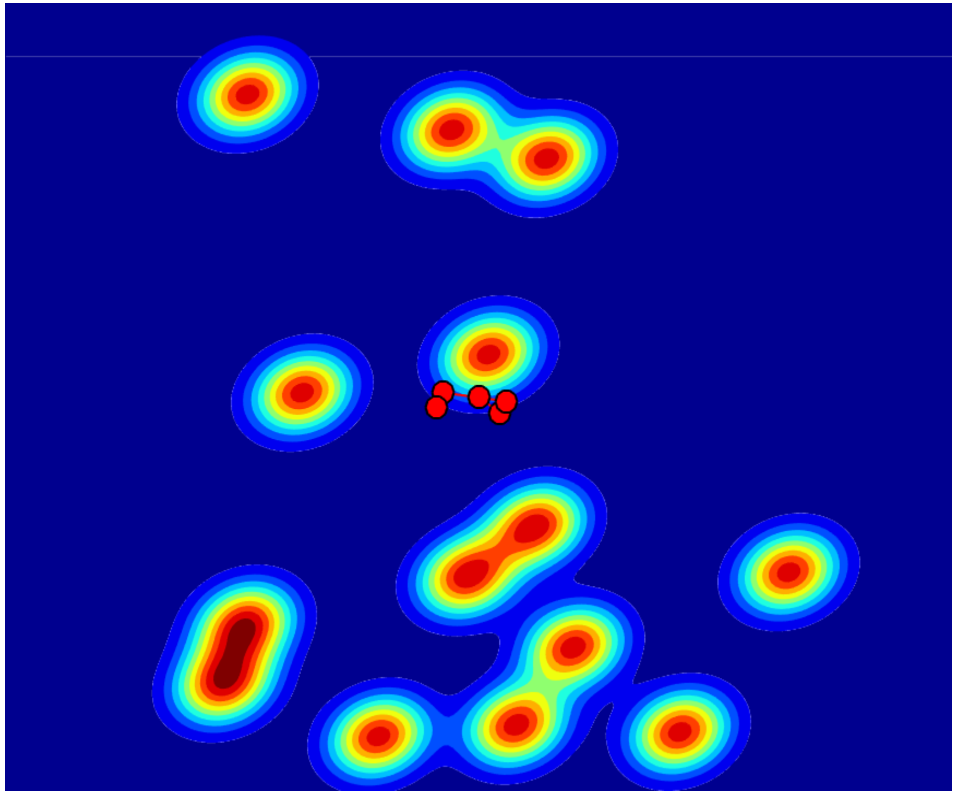
Algorithm Metropolis-coupled Markov chain Monte Carlo

Choose a starting point x_1.
for $t = 2$ to L **do**
    for $m = 1$ to M **do** ▷ update all M chains
        Draw a candidate sample $x' \sim \mathcal{N}(x_{t-1}^m, \sigma)$ ▷ Gaussian proposal distribution
        Sample $u \sim U[0,1]$
        $A^m = \min\{1, [\frac{\pi(x')}{\pi(x_{t-1}^m)}]^{1/T_m}\}$
        if $u < A^m$ **then** $x_t^m = x'$ **else** $x_t^m = x_{t-1}^m$ **end if** ▷ Metropolis acceptance rule
    end for
    repeat floor($M/2$) **times** ▷ swapping scheme for Markov chains
        Randomly select two chain i, j without repetition
        Sample $u \sim U[0,1]$
        $A^{swap} = \min\{1, \frac{\pi(x_t^j)^{1/T_i}\pi(x_t^i)^{1/T_j}}{\pi(x_t^i)^{1/T_i}\pi(x_t^j)^{1/T_j}}\}$
        if $u < A^{swap}$ **then** swap(x_t^i, x_t^j) **end if** ▷ Metropolis-coupled swapping rule
    end repeat
end for



Geyer (1991); Neal (1996)

Metropolis-coupled Markov Chain Monte Carlo (MC^3)

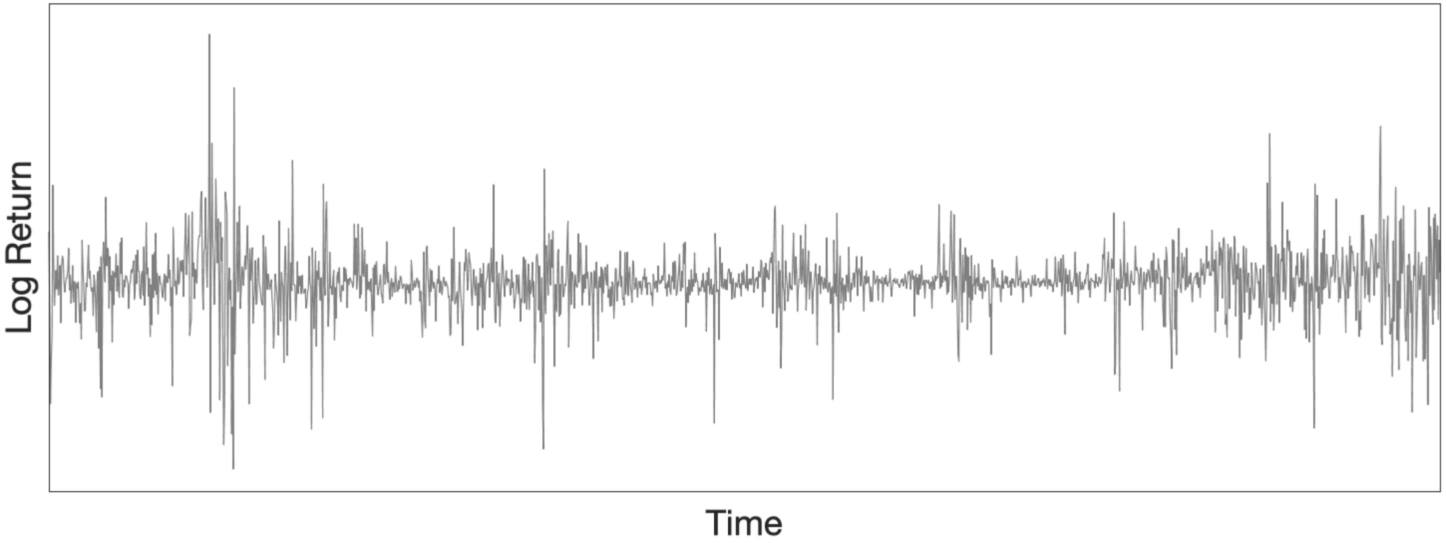


[6 parallel chains (only the cold chain is shown here)]

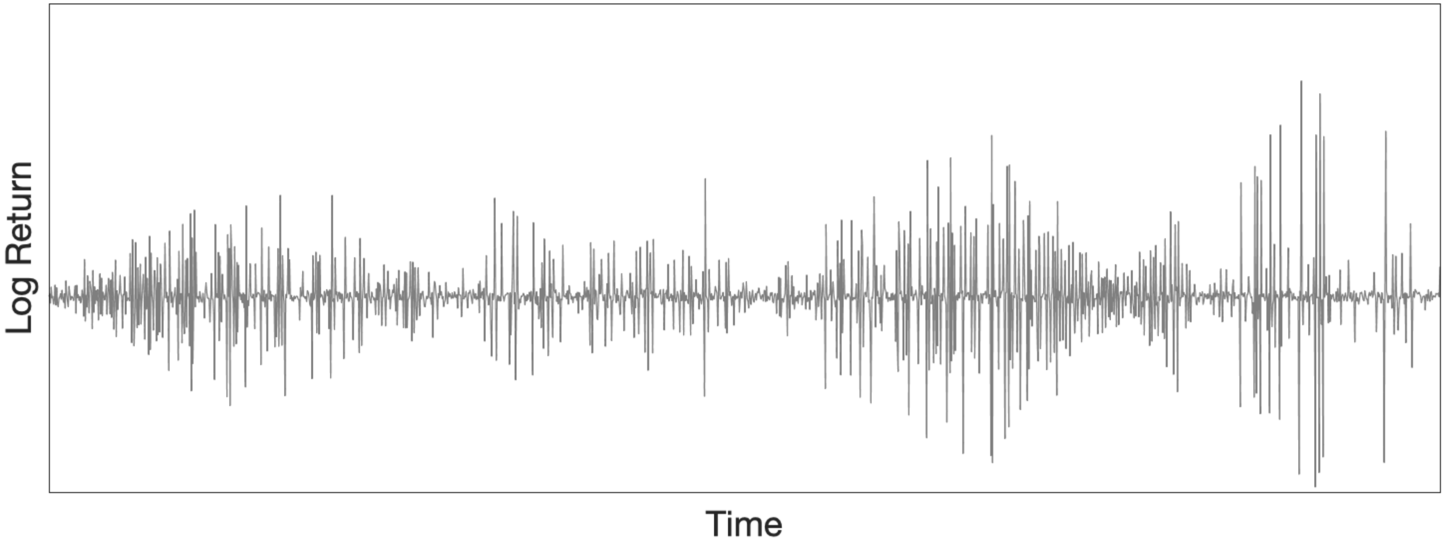
Zhu, Sanborn, & Chater (2018)

Congruent properties of financial time series and MC^3 sampler

Bitcoin/USD exchange rate



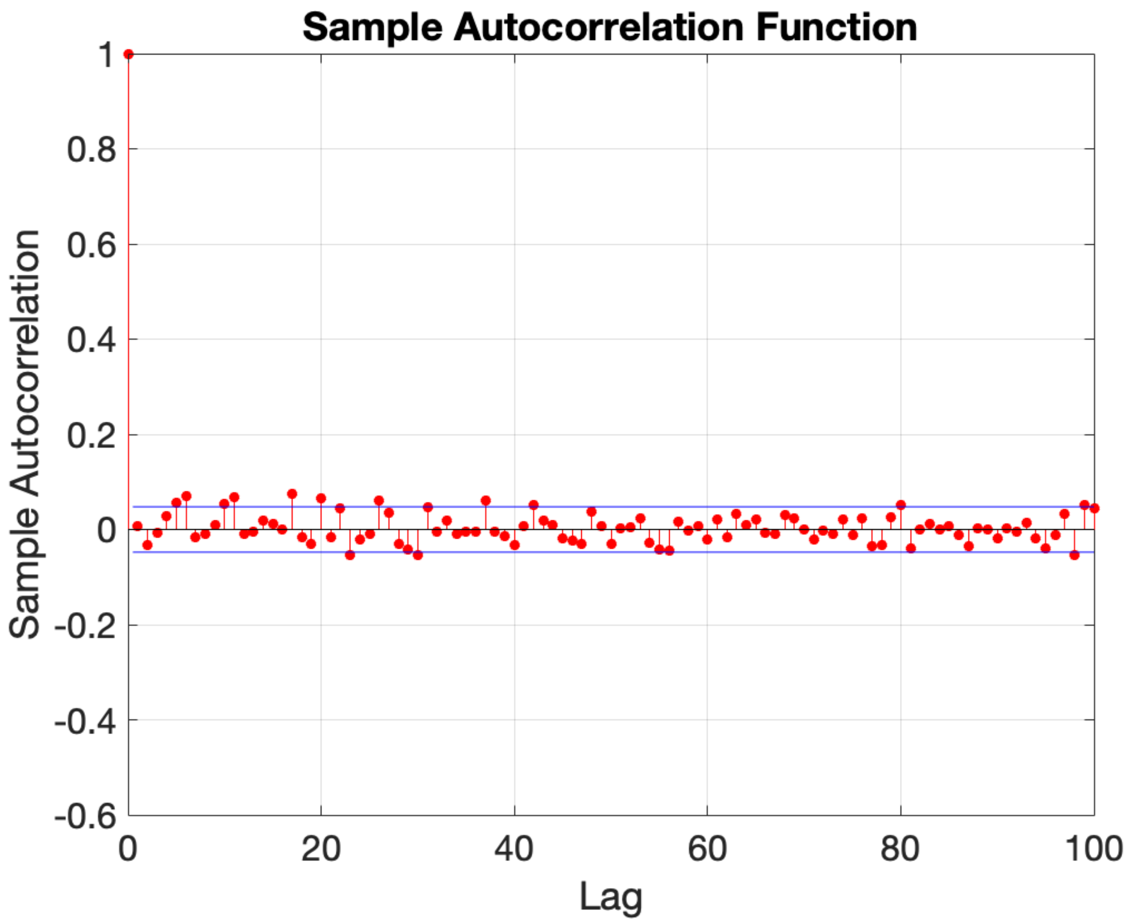
MC^3 sampler



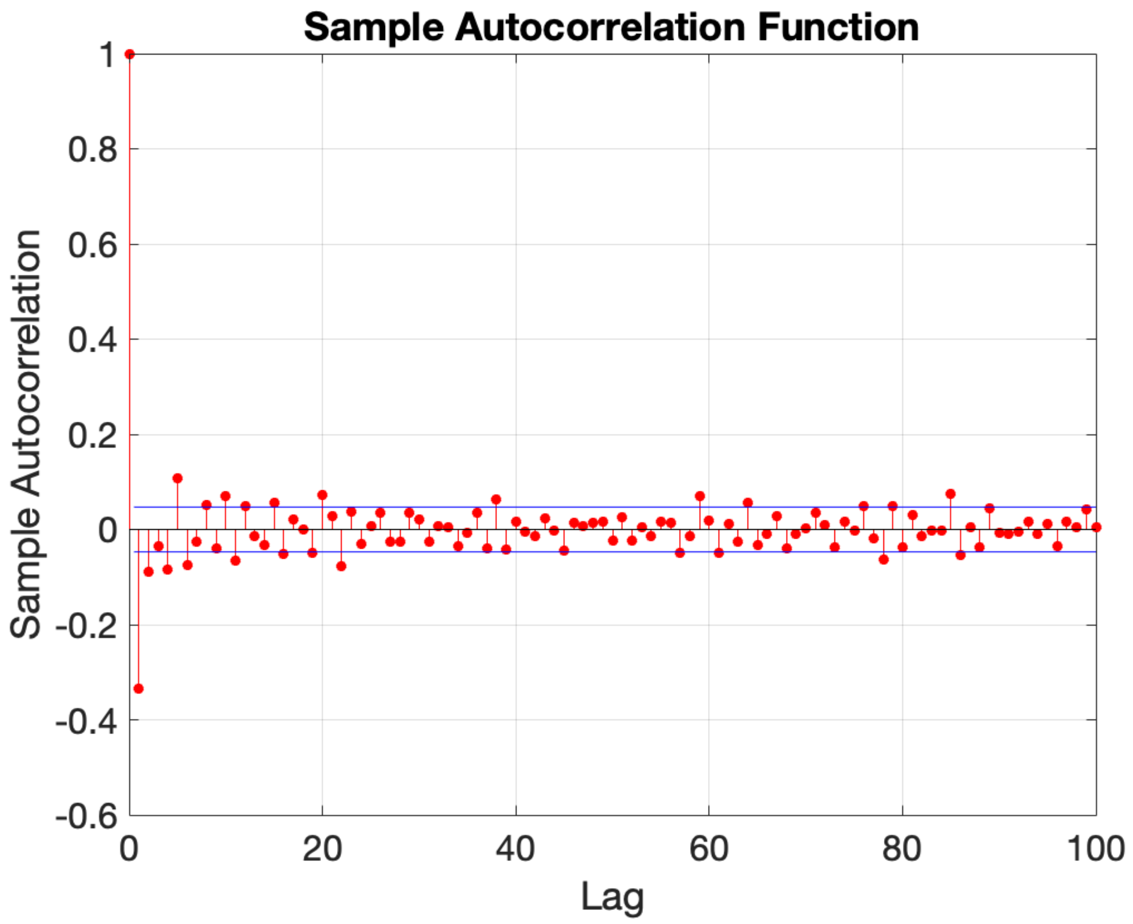
Congruent properties of financial time series and MC³ sampler

[1] Absence of autocorrelation in asset returns

Bitcoin/USD Exchange Rate



MC³ sample

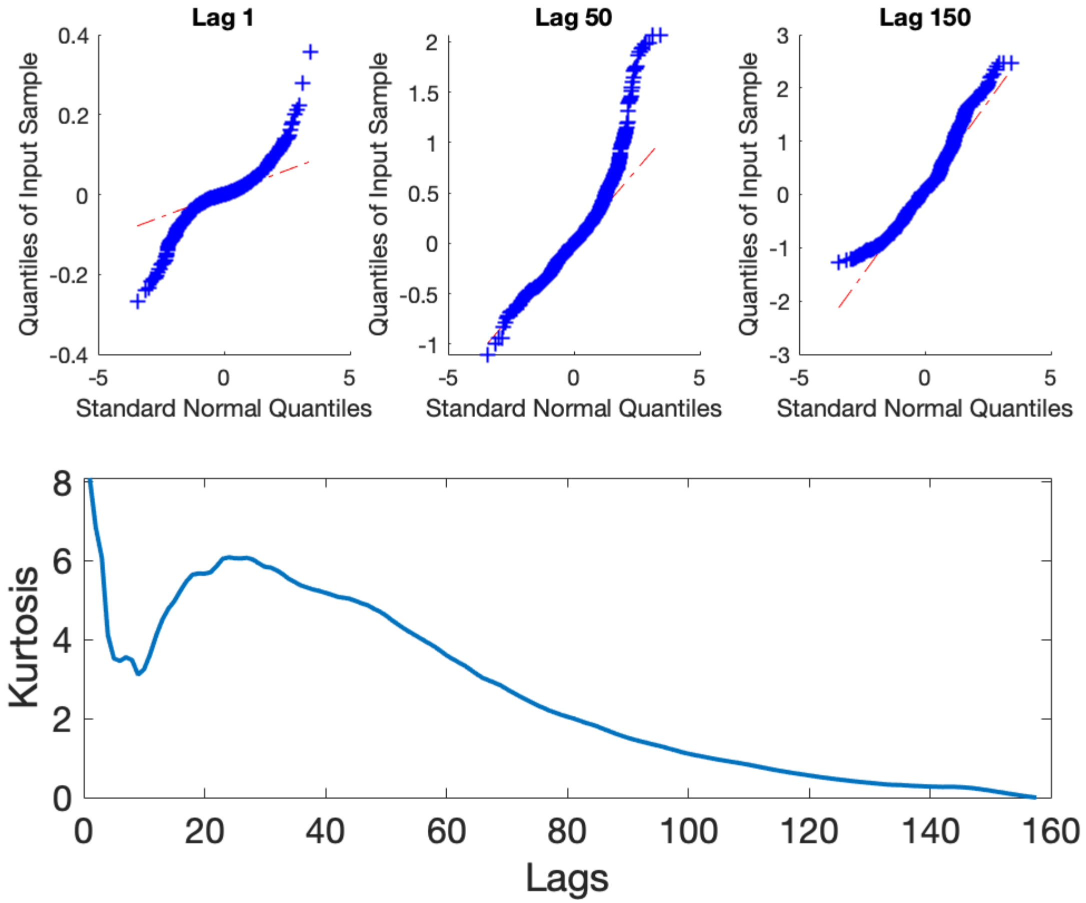


Congruent properties of financial time series and MC³ sampler

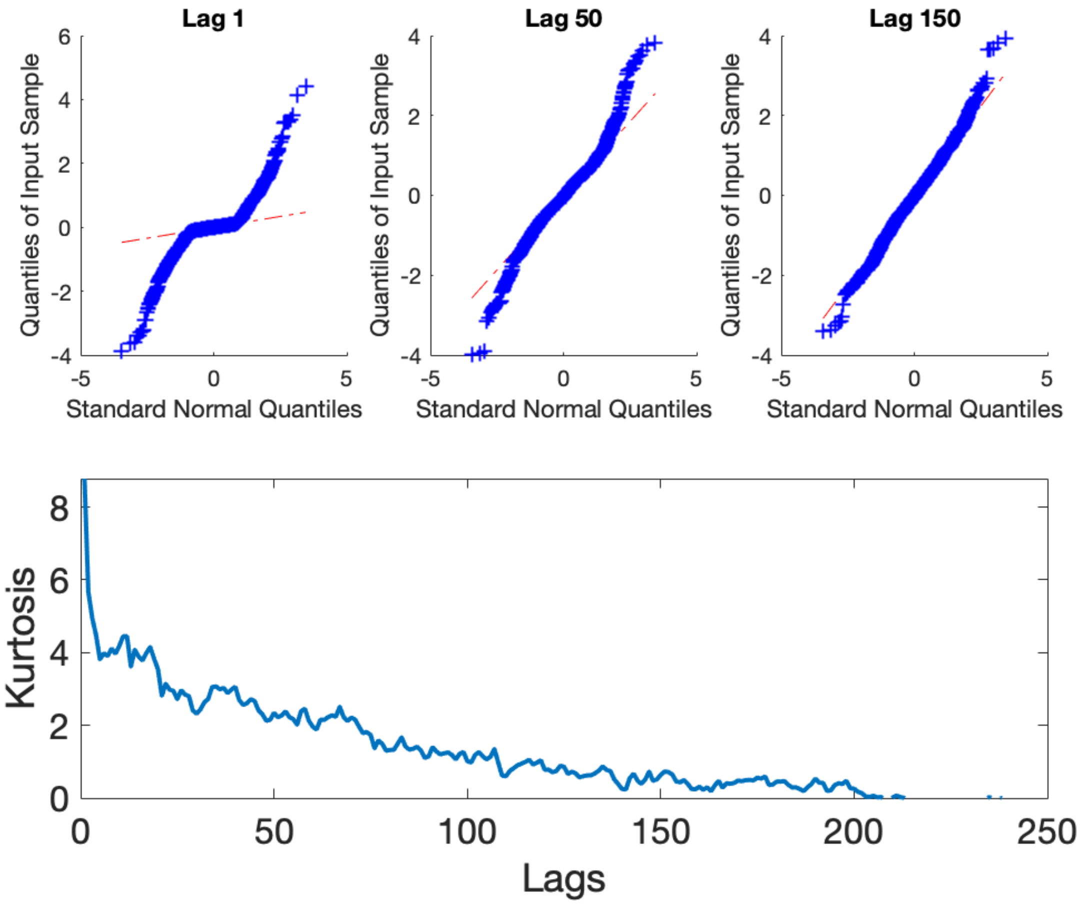
[2] Unconditional heavy tails

[4] Aggregational Gaussianity

Bitcoin/USD Exchange Rate



MC³ sample



Congruent properties of financial time series and MC³ sampler

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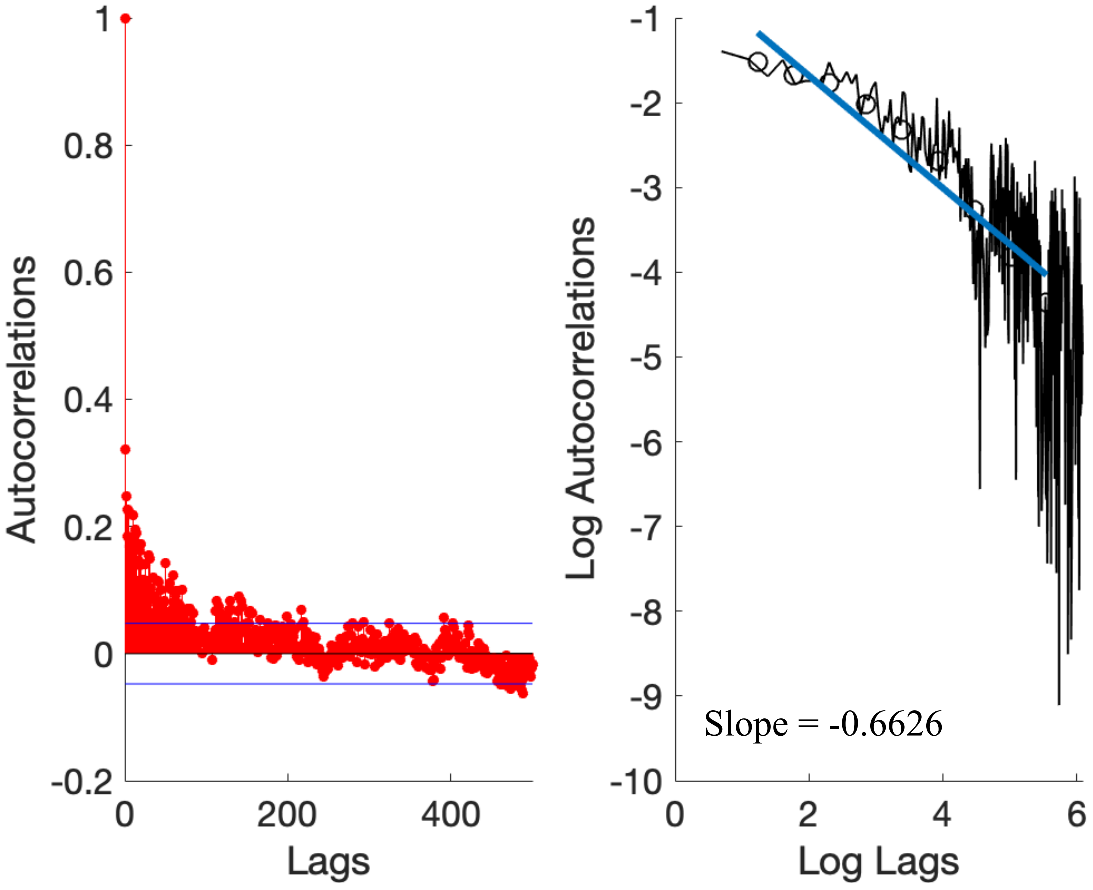
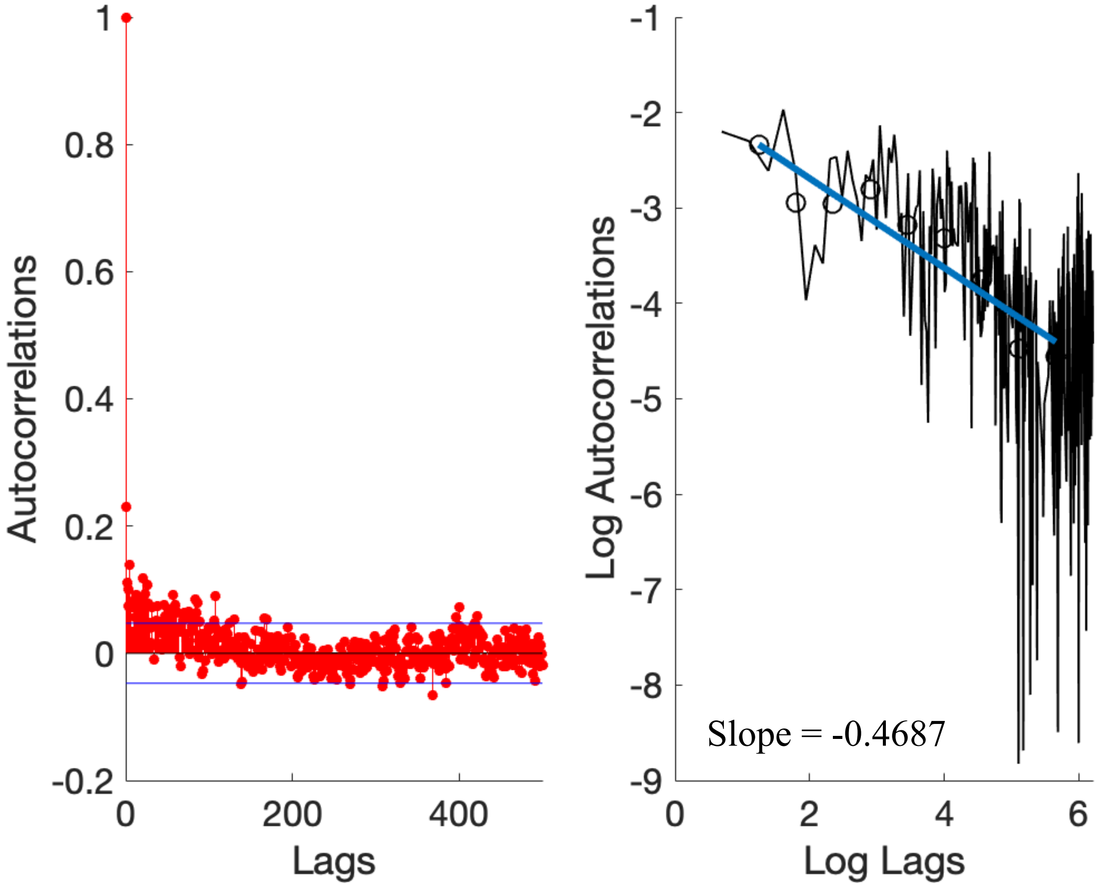
[6] Volatility clustering

[8] Slow decay of autocorrelation in absolute returns

Bitcoin/USD Exchange Rate



MC³ sample

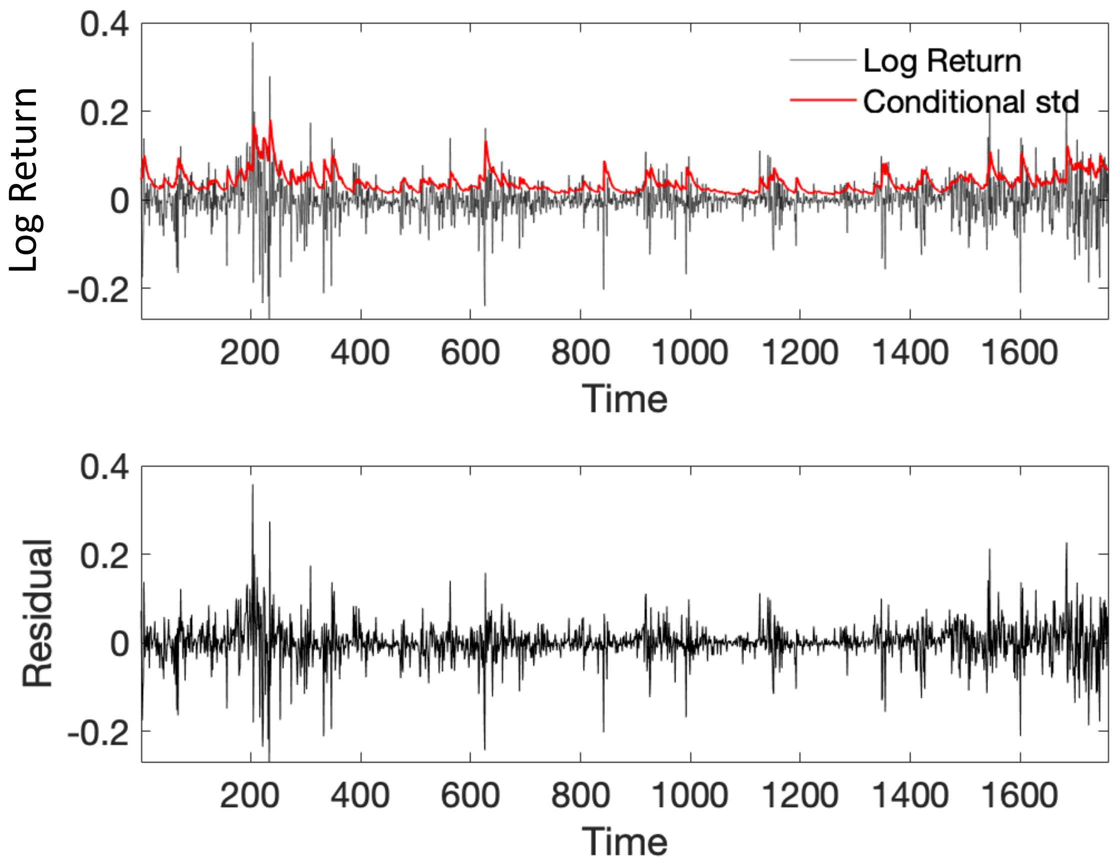


Congruent properties of financial time series and MC³ sampler

[5] Intermittency

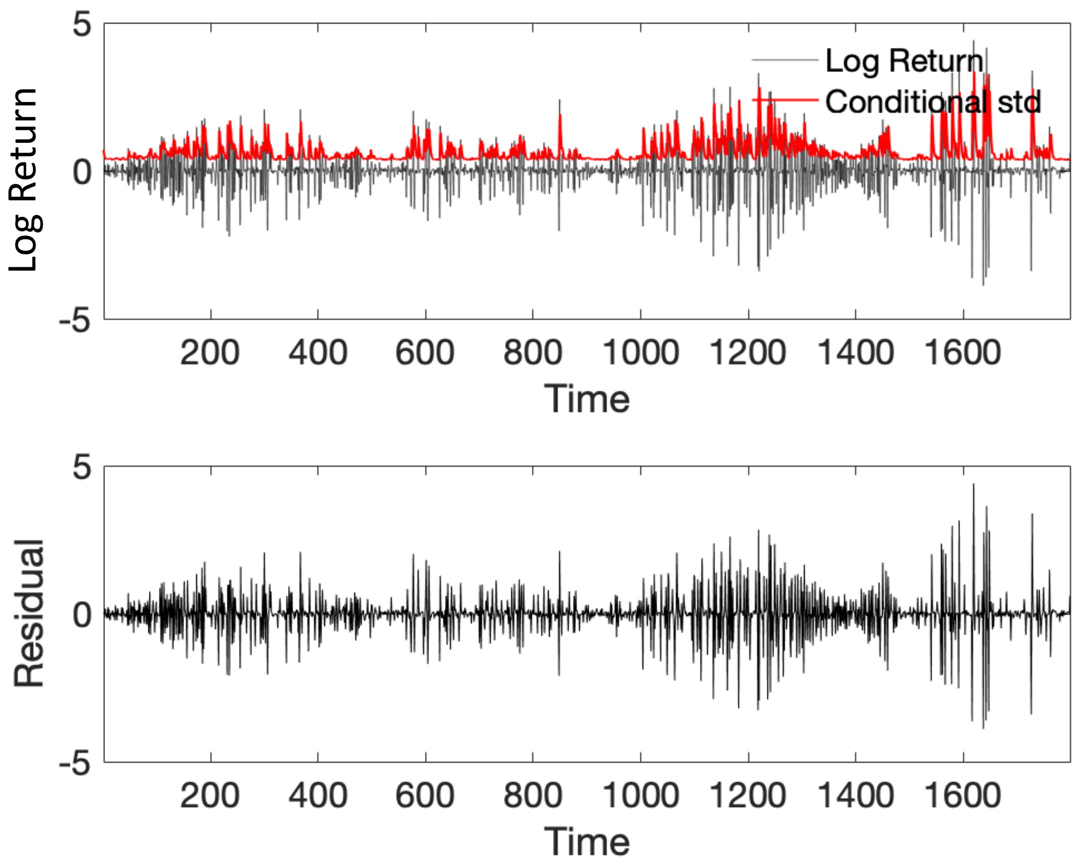
[6] Volatility clustering

Bitcoin/USD Exchange Rate



GARCH(1) = 0.8191 (p<0.0001)

MC³ sample

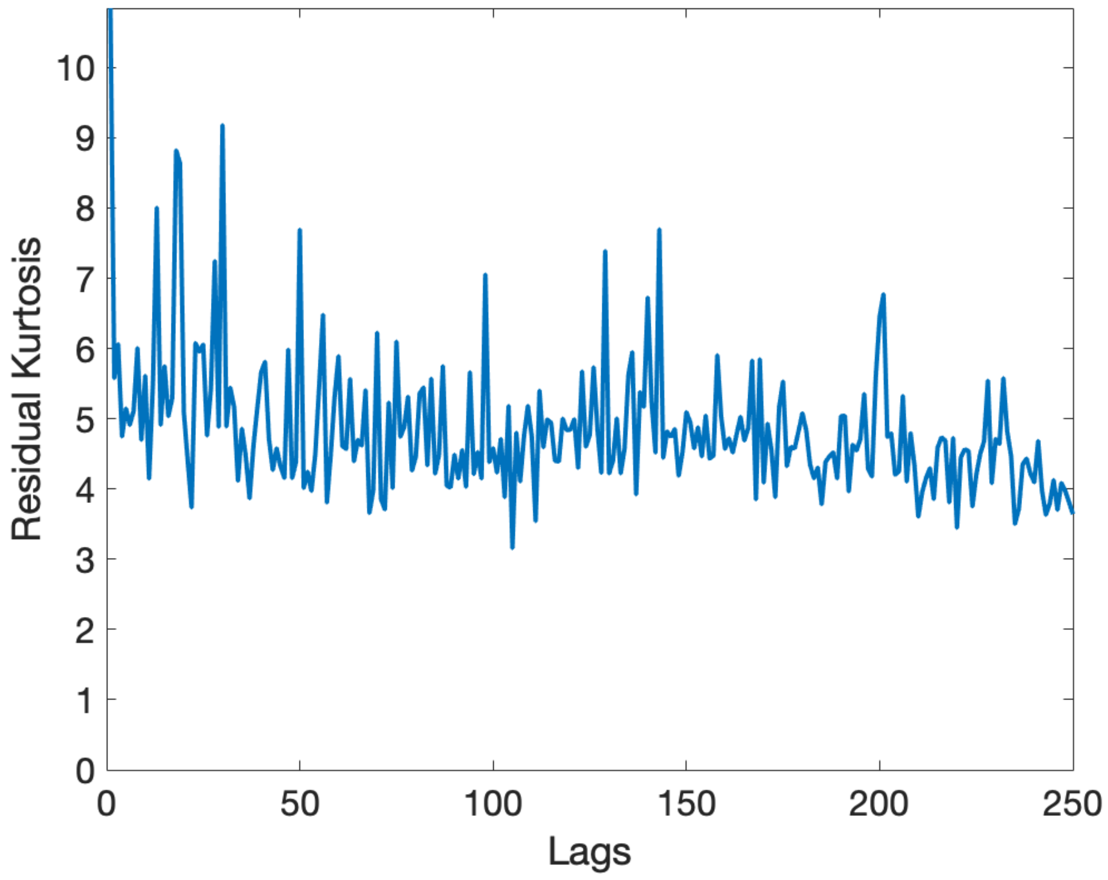


GARCH(1) = 0.4650 (p<0.0001)

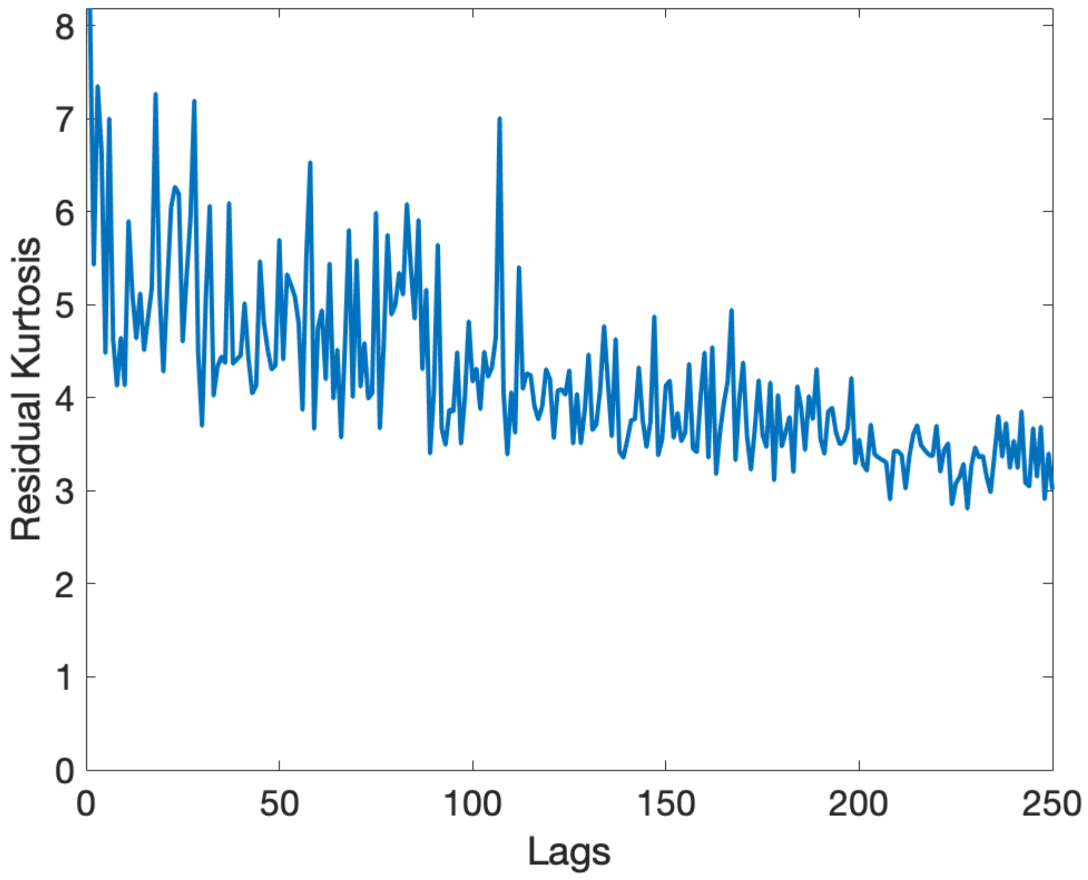
Congruent properties of financial time series and MC³ sampler

[7] Conditional heavy tails

Bitcoin/USD Exchange Rate



MC³ sample



Adding Market Mechanisms

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- To provide market mechanisms, we drew on the model of De Long et al. (1990), where noise traders include a systematic bias sampled from a normal distribution:

$$\rho_t \sim \mathcal{N}(\rho^*, \sigma_\rho^2)$$

- **Market Mechanisms:**
  1. Two-period model (invest young, consume old)
  2. CARA utility function for both traders: $U(w) = -e^{-2\gamma w}$
  3. Risk-free asset: return *r*, infinite supply
  4. Risky asset: dividend *r*, supply = 1
  5. μ noise traders, $1 - \mu$ rational traders

- We can then replace this IID sampler with the MC^3 algorithm to create a new market model.

De Long, Shleifer, Summers, & Waldmann (1990)

Adding Market Mechanisms

- Using the **Pricing function** from the original De Long model (with market clearing & steady-state assumption), price becomes a linear function of the sample:

$$P_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}(\frac{\mu}{1+r})^2\sigma_\rho^2$$

  - Predicted price movements from the edited model therefore match with the movements of the MC^3 algorithm, showing the same behaviours described previously.
  - This does allow us to explore other market behaviours, however, such as volume of trading

- This does however assume that all noise traders hold the same belief, which may not be plausible in a real market
  - We therefore decided to expand the model by dividing noise traders into subgroups with individual beliefs taken from separate samples

De Long, Shleifer, Summers, & Waldmann (1990)

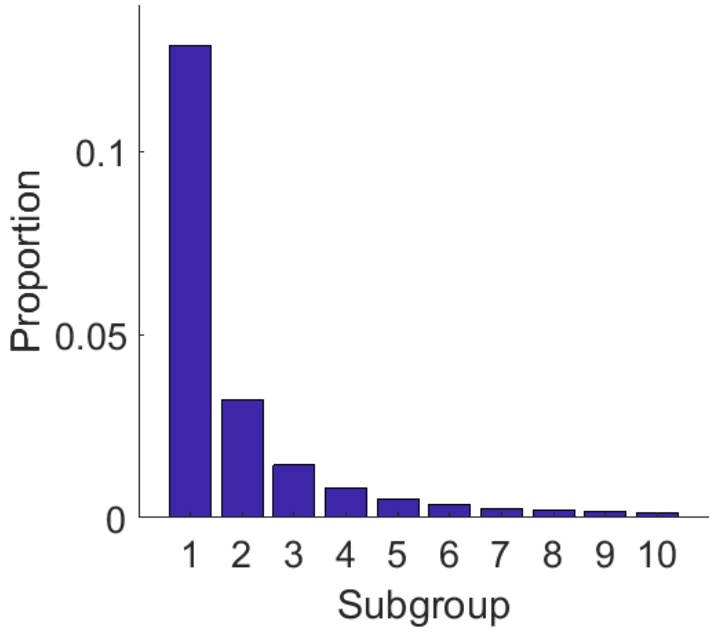
Adding Market Mechanisms

- Noise traders were further divided into a set of subgroups j with proportions μ_j
  - Each subgroup then generates its own sample $\rho_t^{(j)}$ using MC[3], and the aggregate across these samples is used to generate the price:

$$P_t = 1 + \frac{\sum_j \mu_j(\rho_t^{(j)} - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}(\frac{\mu}{1+r})^2 {\sigma_\rho}^2$$

- Under equal proportions, price essentially averages across samples, so many of the previously described behaviours are lost

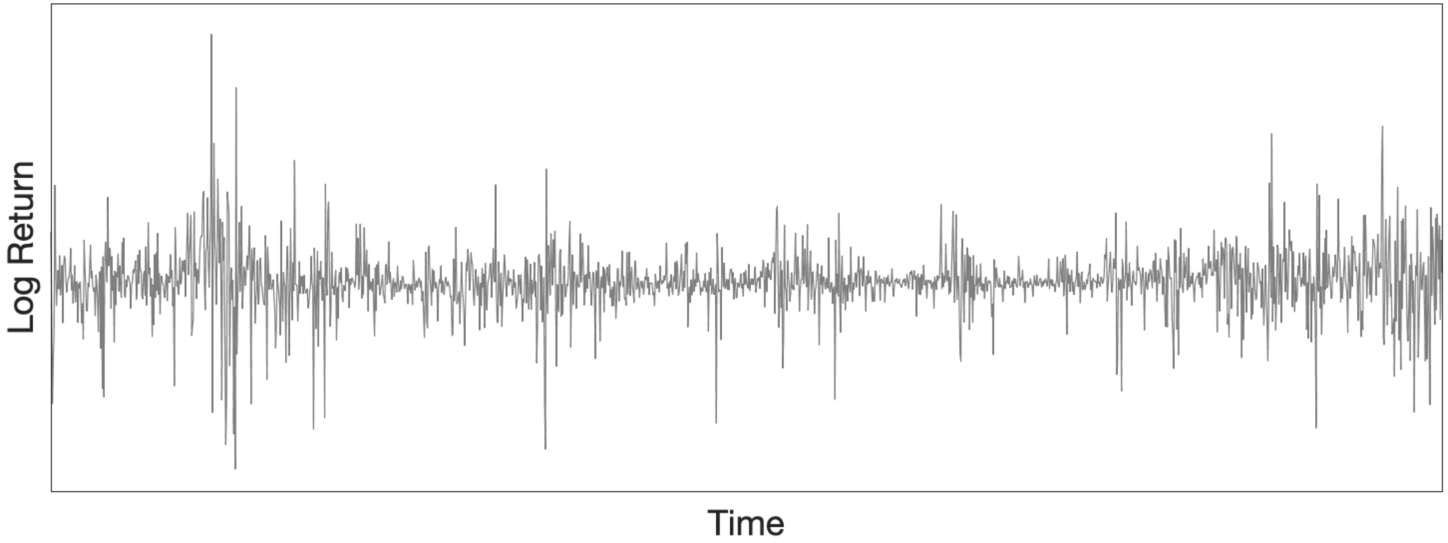
- If we instead assume that some beliefs are more common than others, the some samples can dominate market movements
  - We can represent this using a power law across subgroup proportions, based on similar patterns in scale-free networks



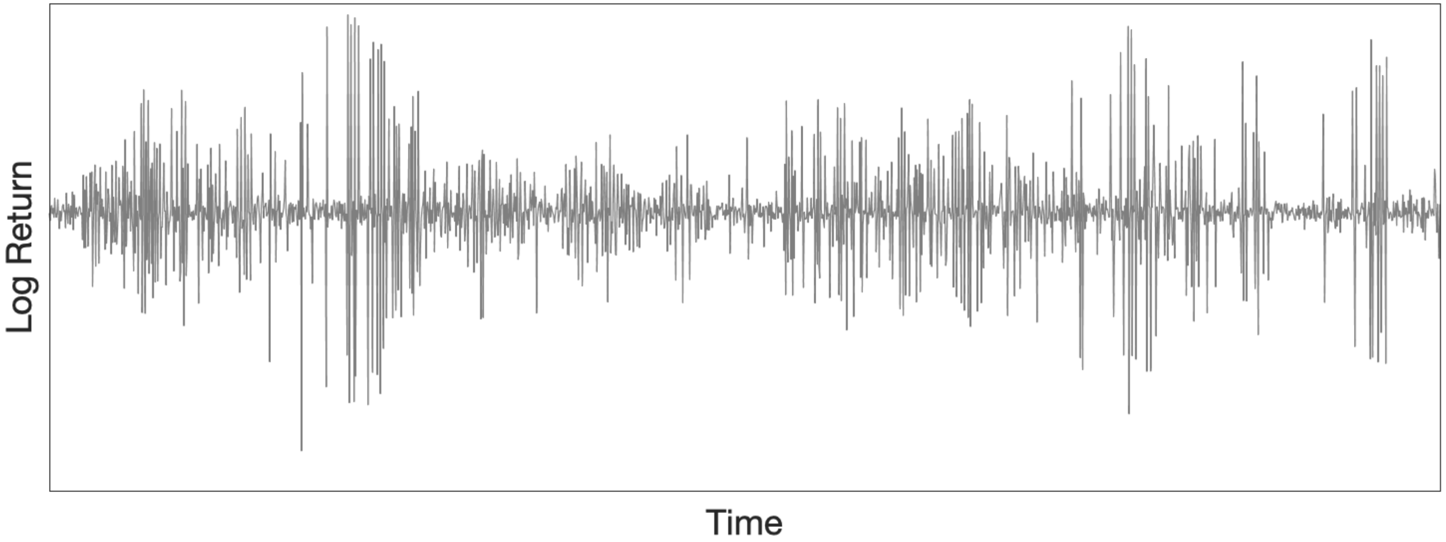
Congruent properties of financial time series and MC³ market model

[1] Absence of autocorrelation in asset returns

Bitcoin/USD exchange rate



MC³ + noise trader model



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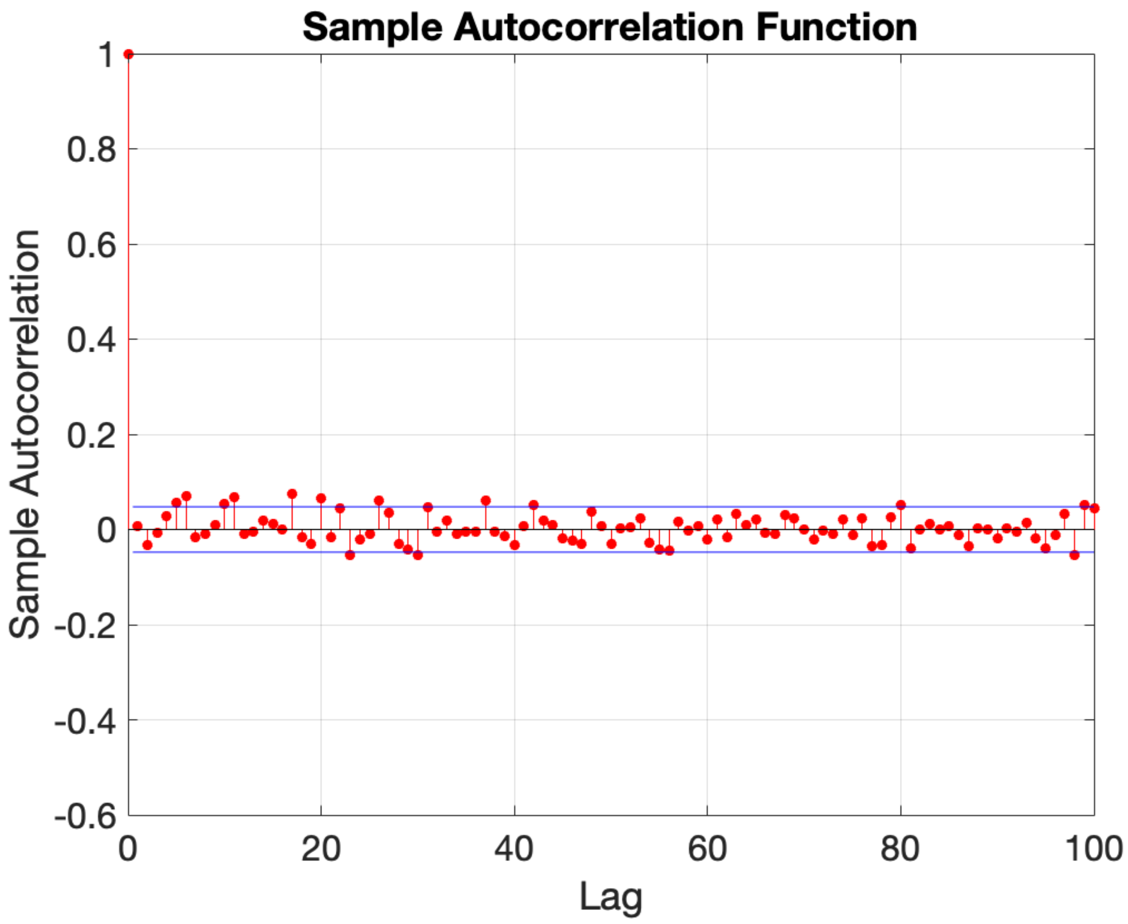
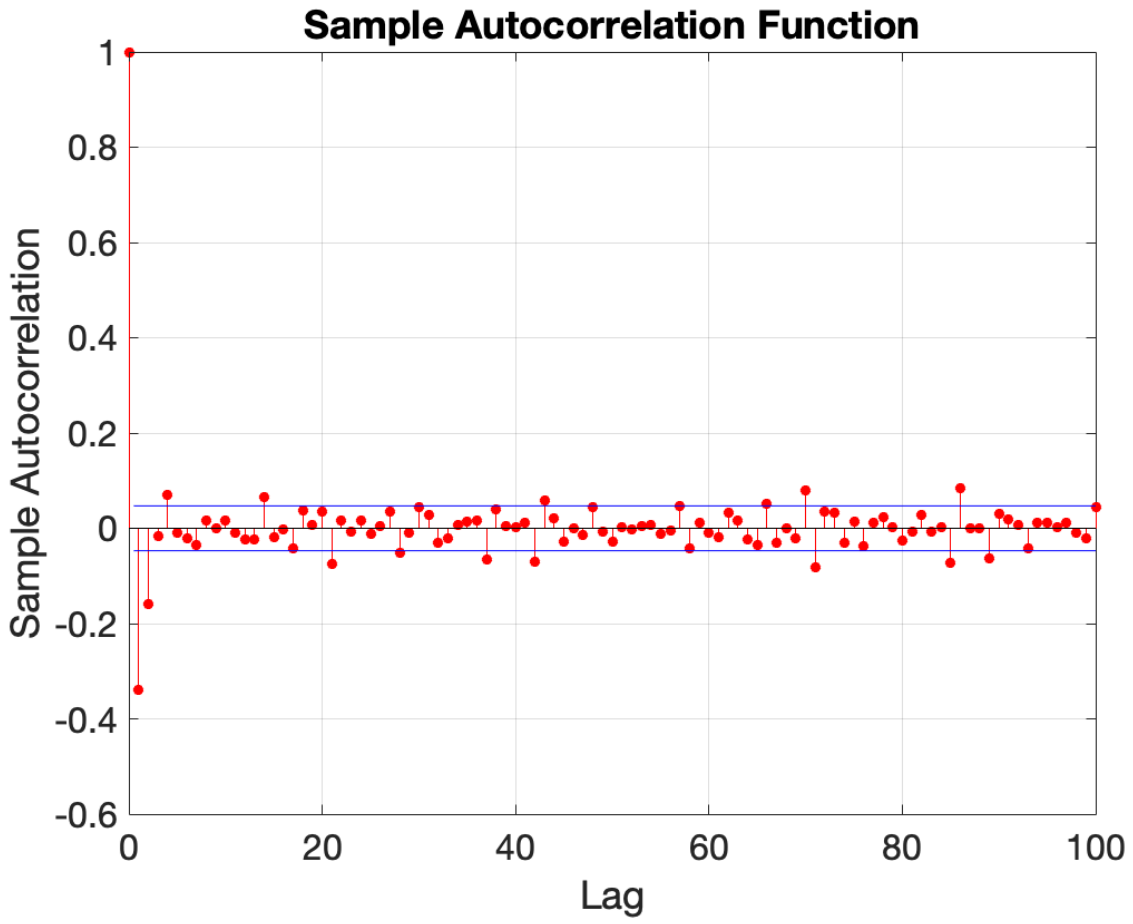


Noise trader model + MC3



Congruent properties of financial time series and MC³ market model

[2] Unconditional heavy tails

[4] Aggregational Gaussianity

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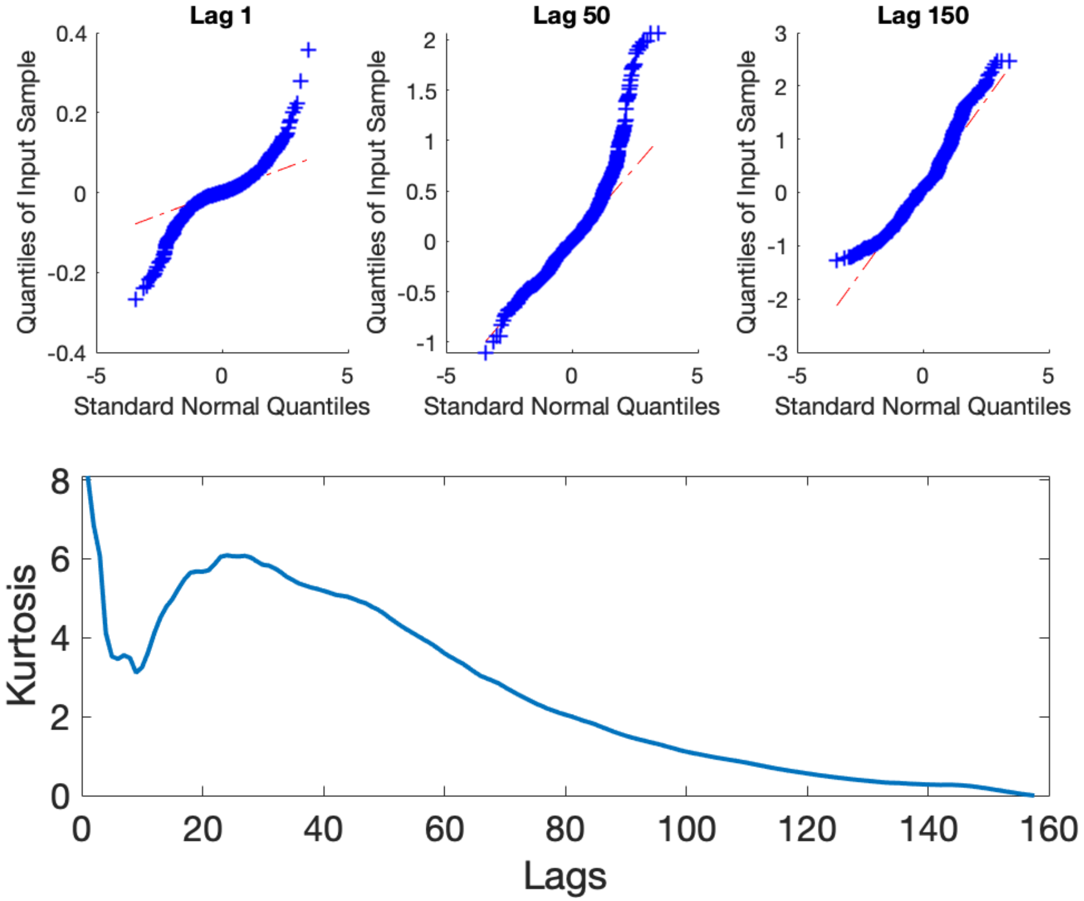
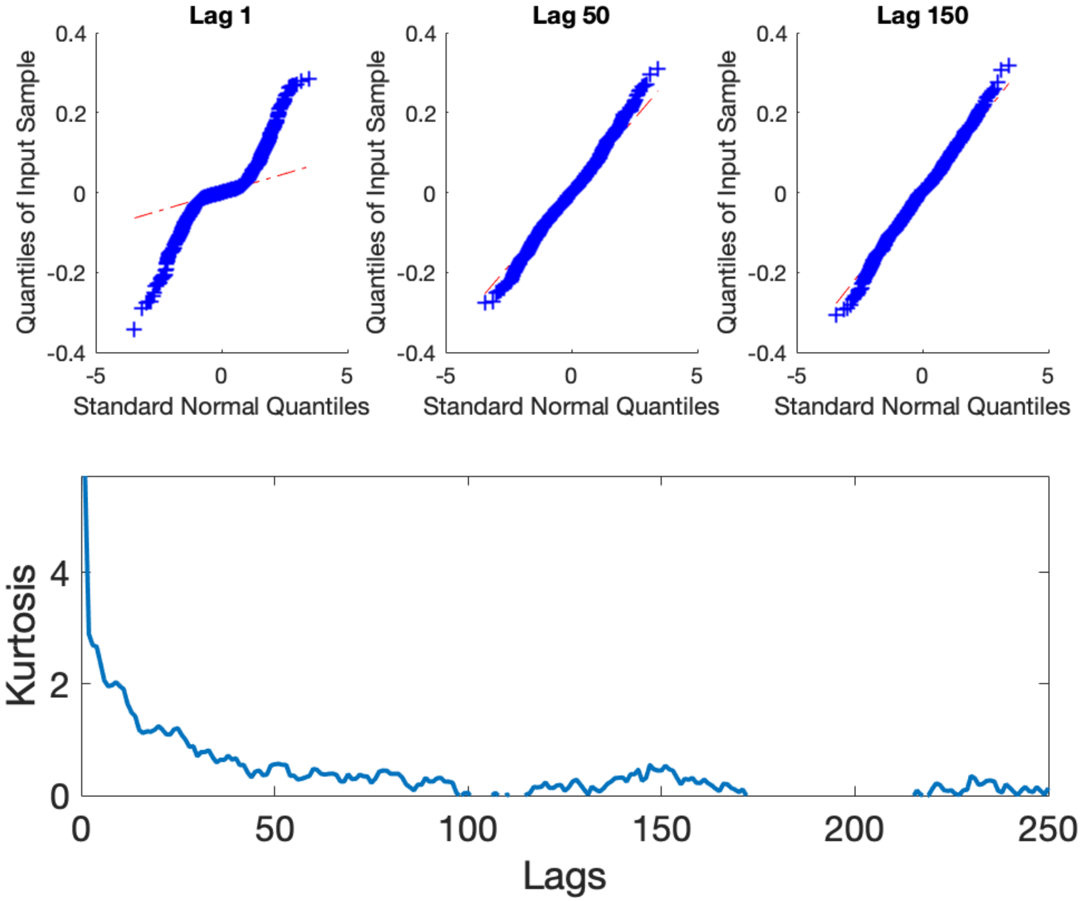
Bitcoin/USD Exchange Rate



Noise trader model + MC3

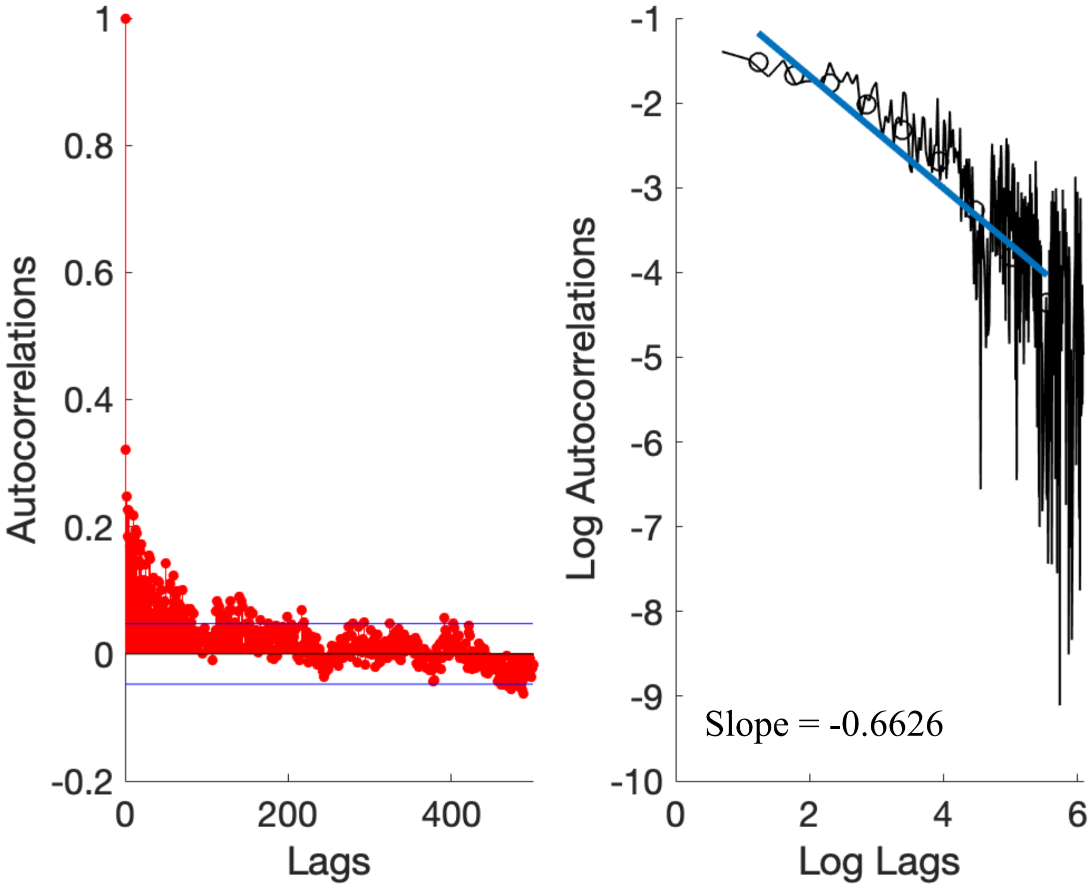


Congruent properties of financial time series and MC³ market model

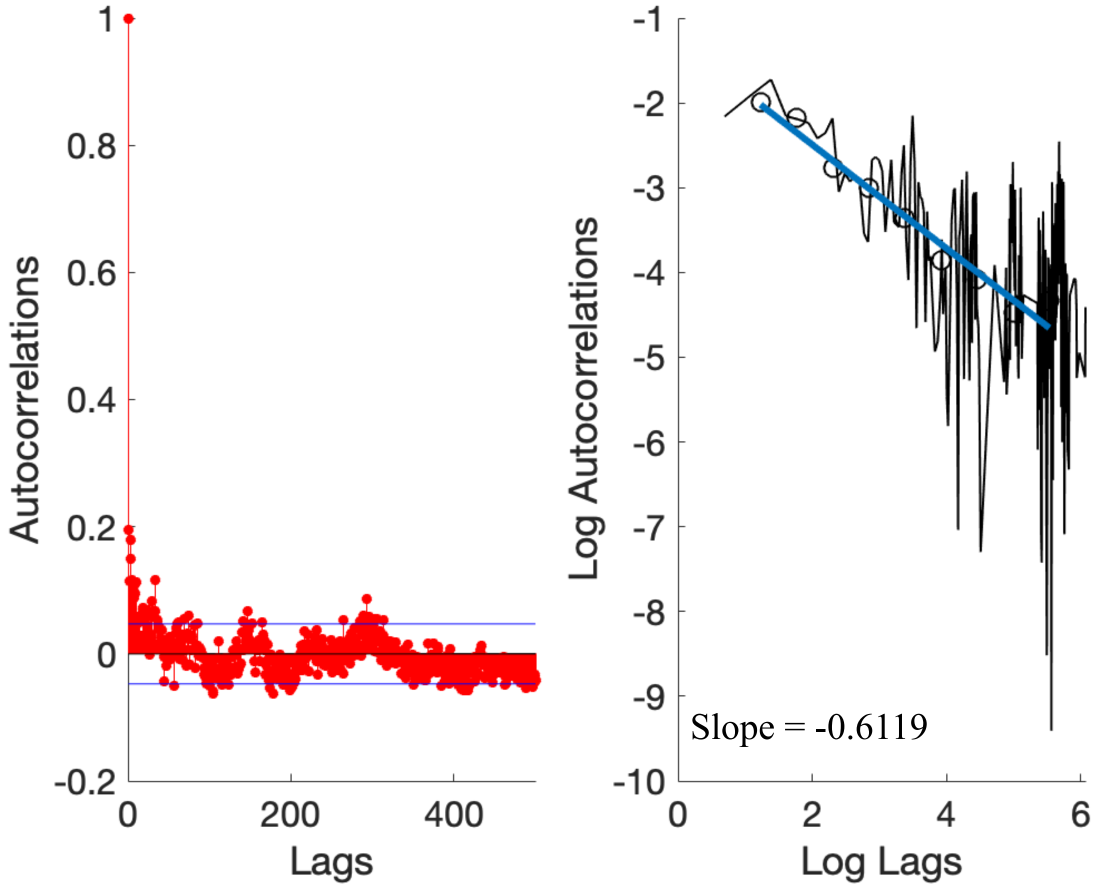
[6] Volatility clustering

[8] Slow decay of autocorrelation in absolute returns

Bitcoin/USD Exchange Rate



Noise trader model + MC3

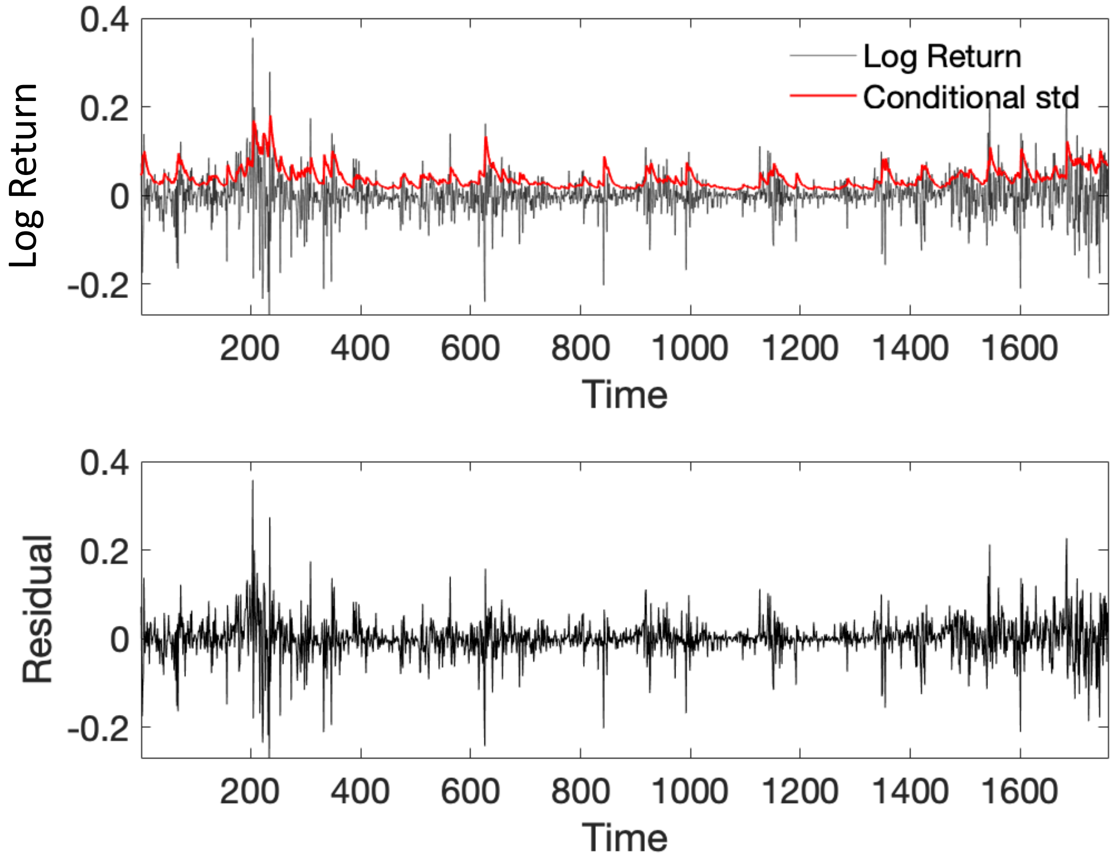


Congruent properties of financial time series and MC³ market model

[5] Intermittency

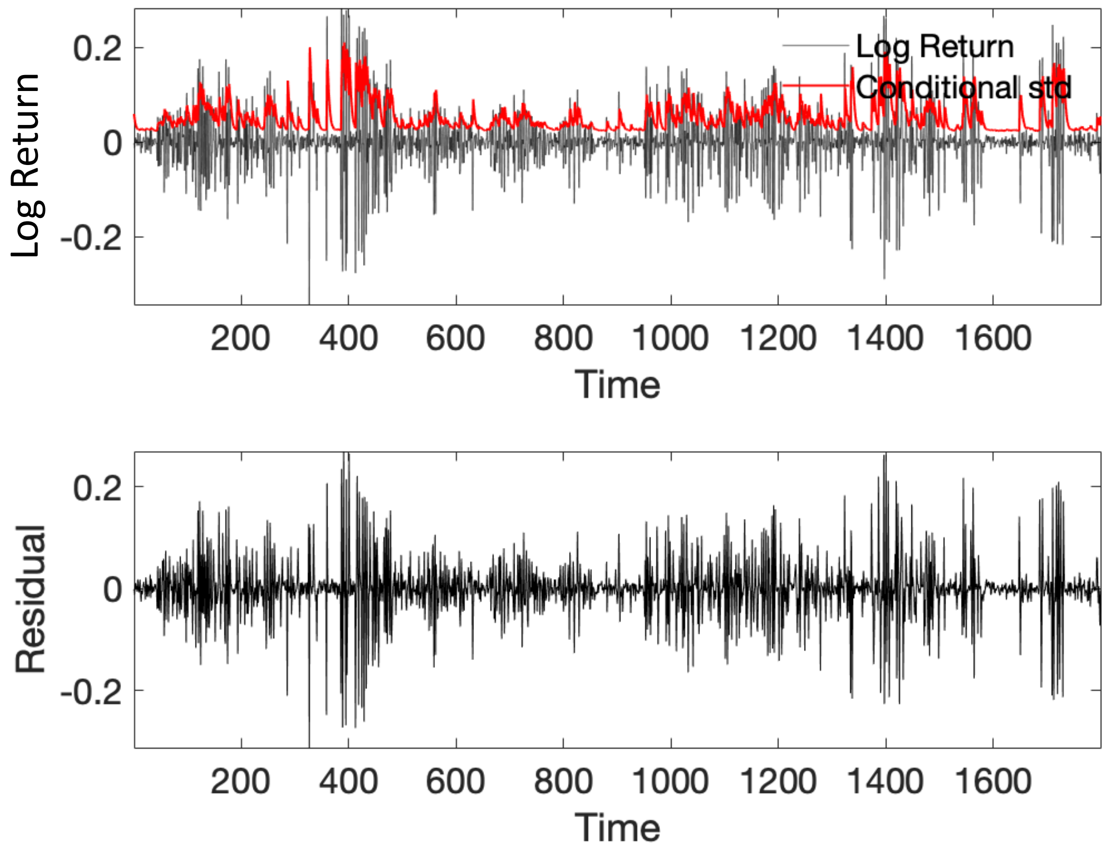
[6] Volatility clustering

Bitcoin/USD Exchange Rate



GARCH(1) = 0.8191 (p<0.0001)

Noise trader model + MC3

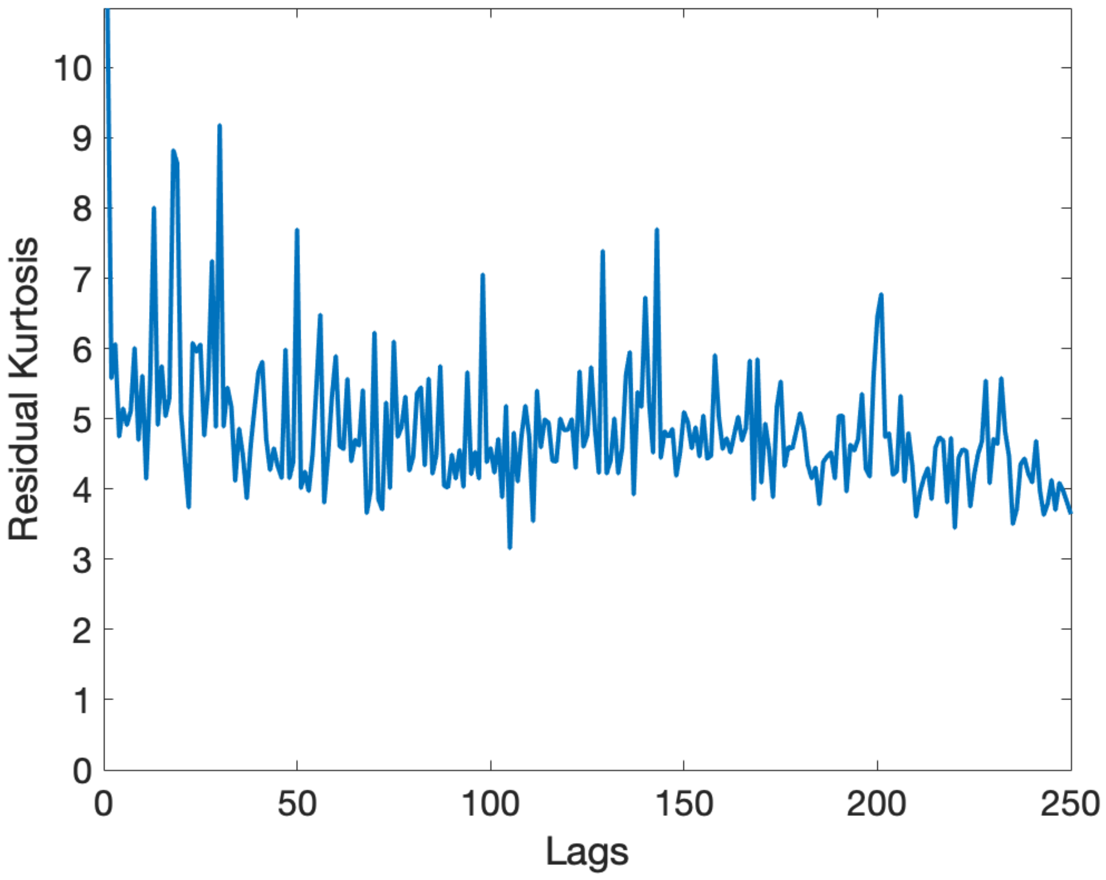


GARCH(1) = 0.6322 (p<0.0001)

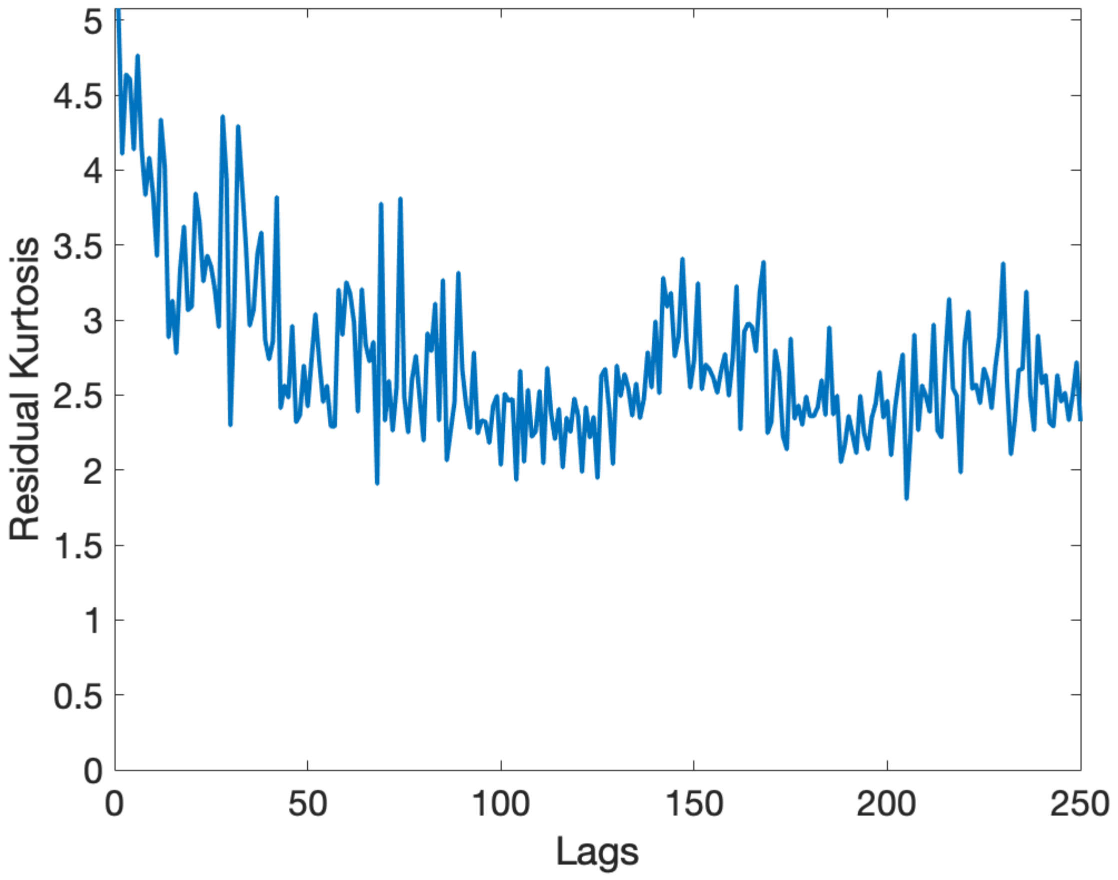
Congruent properties of financial time series and MC³ market model

[7] Conditional heavy tails

Bitcoin/USD Exchange Rate



Noise trader model + MC3

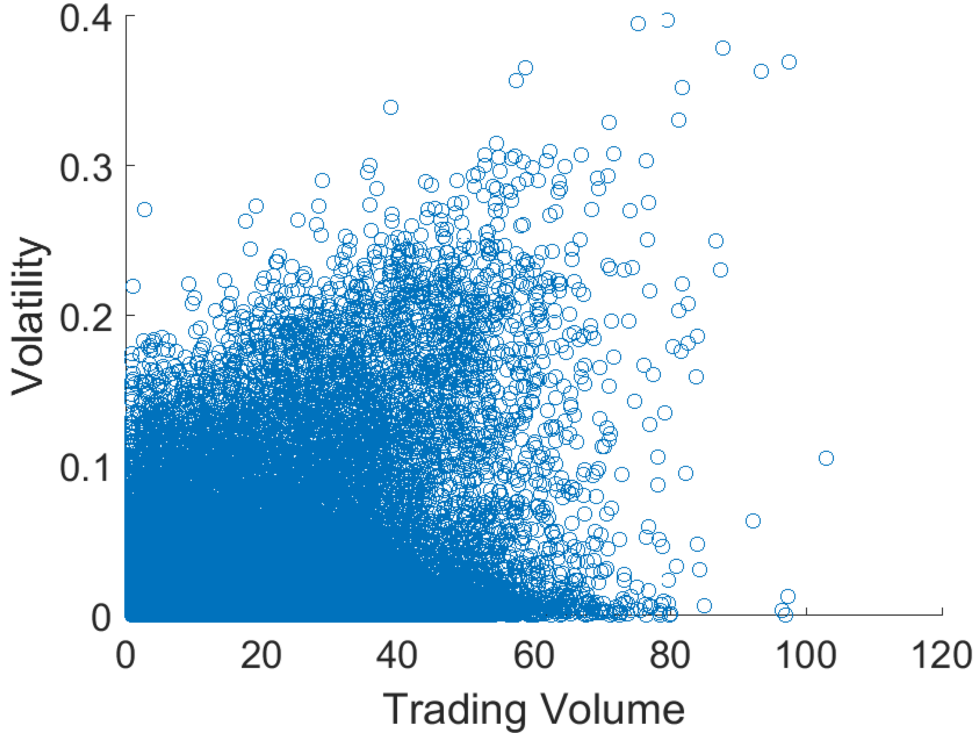


Congruent properties of financial time series and MC^3 market model

[10] Volume/Volatility Correlation

- The De Long model provides an analogue for trading volume in the demands of its two trader types
  - Traders seek to purchase an amount of the asset which maximises their utility given their beliefs
  - By summing the absolute value of these demands, weighted by their proportion in the population, we can compare trading volume against volatility
  - This shows a moderate positive correlation, meeting the criterion of Cont (2001)

Noise trader model + MC3



r = 0.256

Model Limitations

- While the MC^3 noise trader model does match with many of the behaviours of financial markets, there are others which are not possible in our current framework:
  - **Gain/loss asymmetry** – Markets display bigger drawdowns than upwards movements
    - Our model allows for extreme movements in both directions
  - **Leverage effect** – Falls in return are correlated with increases in volatility at short time lags
    - This may require some measure of 'panic' from falling prices
  - **Scale asymmetry** – Coarse-grained measures of volatility predict fine-grained scales better than fine predict coarse
    - Our model includes no direction of time, so predictions are symmetrical

- Additional psychological mechanisms could be added to the model to capture these behaviours e.g. loss aversion

Summary

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- The movements of financial markets bear a strong resemblance to individual decisions
- Sampling algorithms such as MC^3 are able to capture many of these behaviours at both an individual and market level
- By combining such sampling methods with existing financial theories, we have developed a model which accurately predicts many of the complex behaviours of financial markets
- But there is still room for expansion

Thank you